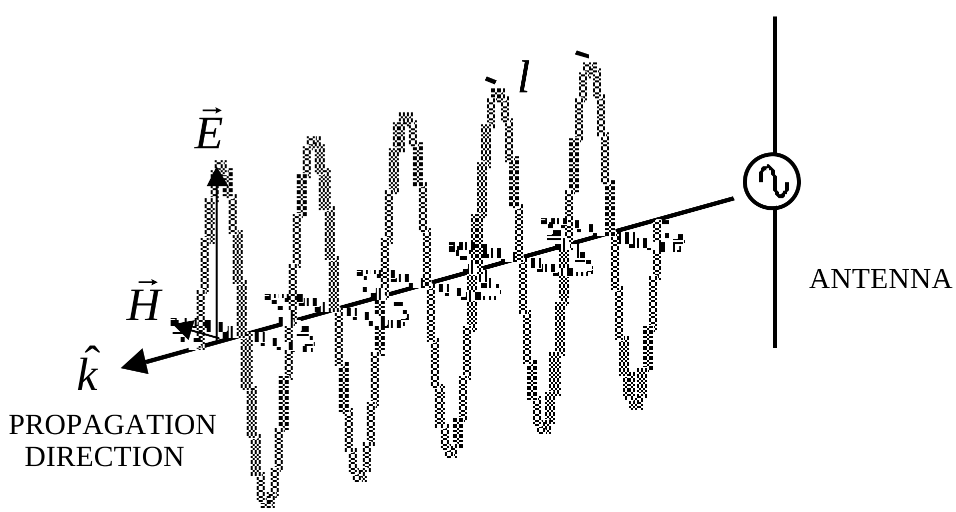
Antennas & Propagation

LECTURE NOTES
VOLUME II

BASIC ANTENNA PARAMETERS AND WIRE ANTENNAS

by Professor David Jenn



(*ver 1.3*)

Antennas: Introductory Comments

Classification of antennas by size:

Let ℓ be the antenna dimension:

1. electrically small, $\ell << l$: primarily used at low frequencies where the wavelength is long
2. resonant antennas, $\ell \approx l\ /2$: most efficient; examples are slots, dipoles, patches
3. electrically large, $\ell >> l$: can be composed of many individual resonant antennas; good for radar applications (high gain, narrow beam, low sidelobes)

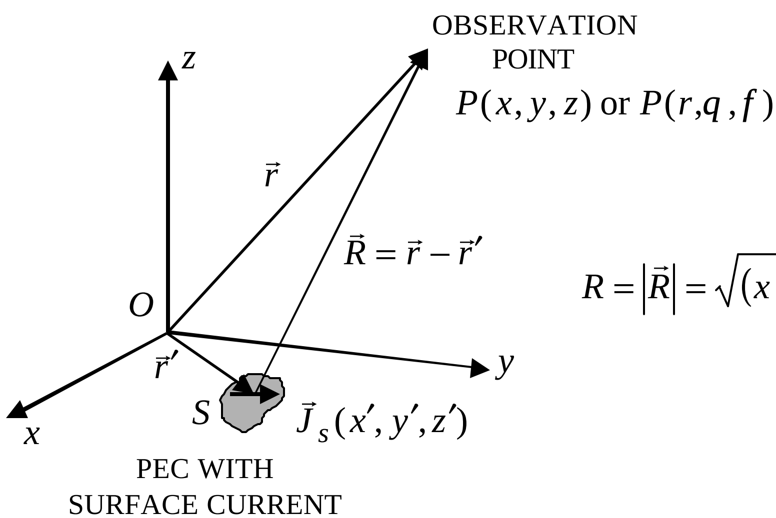
Classification of antennas by type:

1. reflectors
2. lenses
3. arrays

Other designations: wire antennas, aperture antennas, broadband antennas

Radiation Integrals (1)

Consider a perfect electric conductor (PEC) with an electric surface current flowing on *S*. In the case where the conductor is part of an antenna (a dipole), the current may be caused by an applied voltage, or by an incident field from another source (a reflector). The observation point is denoted by *P* and is given in terms of unprimed coordinate variables. Quantities associated with source points are designated by primes. We can use any coordinate system that is convenient for the particular problem at hand.



$$R=\left|\vec{R}\right|=\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}$$

The medium is almost always free space (μ_o,ε_o), but we continue to use (μ,ε) to cover more general problems. If the currents are known, then the field due to the currents can be determined by integration over the surface.

Radiation Integrals (2)

The vector wave equation for the electric field can be obtained by taking the curl of Maxwell's first equation:

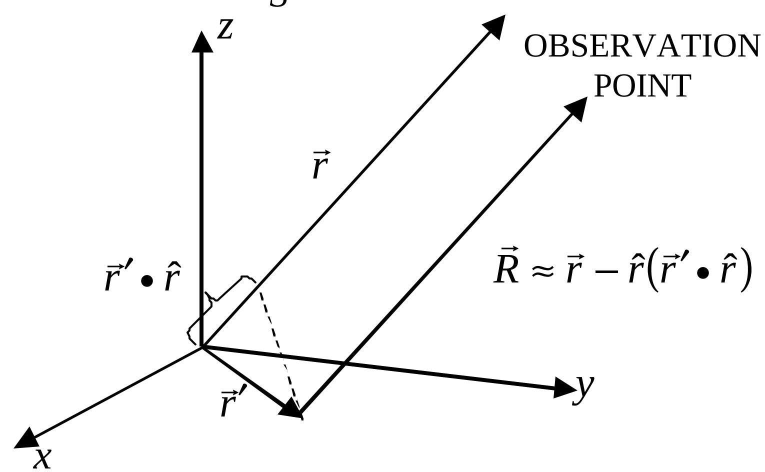
$$\nabla \times \nabla \times \vec{E} = k^2 \vec{E} - jwm\vec{J}_s$$

A solution for $\vec{E}$ in terms of the magnetic vector potential $\vec{A}(\vec{r})$ is given by

$$\vec{E}(\vec{r}) = -jw\vec{A}(\vec{r}) + \frac{\nabla\left(\nabla \bullet \vec{A}(\vec{r})\right)}{jwme} \qquad (1)$$

where $(\vec{r})$ is a shorthand notation for (x,y,z) and $\vec{A}(\vec{r}) = \dfrac{m}{4p} \iint_S \dfrac{\vec{J}_s}{R} e^{-jkR} ds'$

We are particularly interested in the case were the observation point is in the far zone of the antenna ($P \rightarrow \infty$). As P recedes to infinity, the vectors $\vec{r}$ and $\vec{R}$ become parallel.



Radiation Integrals (3)

In the expression for $\vec{A}(\vec{r})$ we use the approximation $1/R \approx 1/r$ in the denominator and $\hat{r} \bullet \vec{R} \approx \hat{r} \bullet [\vec{r} - \hat{r}(\vec{r}\,' \bullet \hat{r})]$ in the exponent. Equation (2) becomes

$$\vec{A}(\vec{r}) \approx \frac{m}{4pr} \iint_S \vec{J}_s \, e^{-jk\hat{r} \bullet [\vec{r} - \hat{r}(\vec{r}\,' \bullet \hat{r})]} ds' = \frac{m}{4pr} e^{-jkr} \iint_S \vec{J}_s \, e^{jk(\vec{r}\,' \bullet \hat{r})} ds'$$

When this is inserted into equation (1), the del operations on the second term lead to $1/r^2$ and $1/r^3$ terms, which can be neglected in comparison to the $-jw\vec{A}$ term, which depends only on $1/r$. Therefore, in the <u>far field</u>,

$$\vec{E}(\vec{r}) \approx \frac{-jwm}{4pr} e^{-jkr} \iint_S \vec{J}_s \, e^{jk(\vec{r}\,' \bullet \hat{r})} ds' \quad \text{(discard the } E_r \text{ component)} \qquad (3)$$

Explicitly removing the r component gives,

$$\vec{E}(\vec{r}) \approx \frac{-jkh}{4p\,r} e^{-jkr} \iint_S \left[\vec{J}_s - \hat{r}\left(\vec{J}_s \bullet \hat{r}\right)\right] e^{jk(\vec{r}\,' \bullet \hat{r})} ds'$$

The radial component of current does not contribute to the field in the far zone.

Radiation Integrals (4)

Notice that the fields have a spherical wave behavior in the far zone: $\left|\vec{E}\right| \sim \frac{e^{-jkr}}{r}$. The spherical components of the field can be found by the appropriate dot products with $\vec{E}$. More general forms of the radiation integrals that include magnetic surface currents ($\vec{J}_{ms}$) are:

$$E_q(r,q,f) = \frac{-jkh}{4p\,r} e^{-jkr} \iint_S \left[\vec{J}_s \bullet \hat{q} + \frac{\vec{J}_{ms} \bullet \hat{f}}{h} \right] e^{jk\vec{r}'\cdot\hat{r}} ds'$$

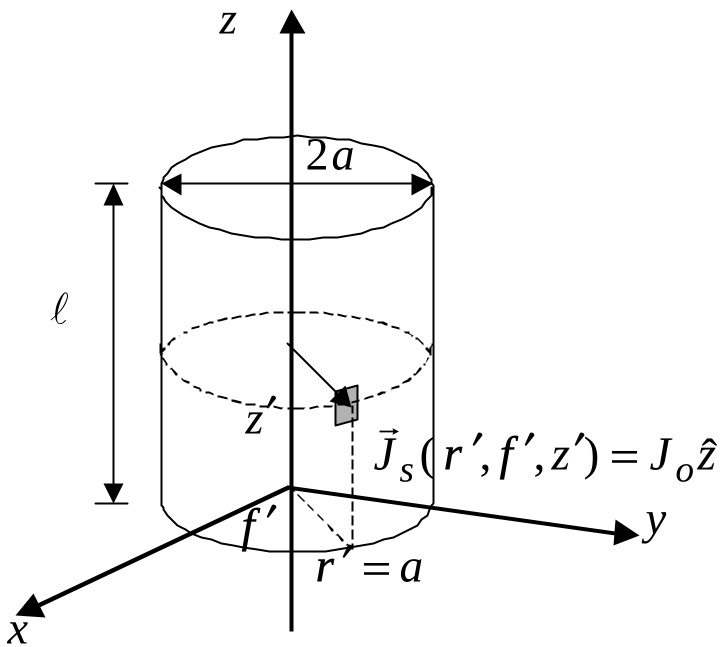
$$E_f(r,q,f) = \frac{-jkh}{4p\,r} e^{-jkr} \iint_S \left[\vec{J}_s \bullet \hat{f} - \frac{\vec{J}_{ms} \bullet \hat{q}}{h} \right] e^{jk\vec{r}'\cdot\hat{r}} ds'$$

The radiation integrals apply to an unbounded medium. For antenna problems the following process is used:

1. find the current on the antenna surface, *S*,
2. remove the antenna materials and assume that the currents are suspended in the unbounded medium, and
3. apply the radiation integrals.

Hertzian Dipole (1)

Perhaps the simplest application of the radiation integral is the calculation of the fields of an <u>infinitesimally short dipole</u> (also called a <u>Hertzian dipole</u>). Note that the criterion for short means much less than a wavelength, which is not necessarily physically short.



- For a thin dipole (radius, $a << l$) the surface current distribution is independent of f'. The current crossing a ring around the antenna is $I = \underbrace{\left|\vec{J}_s\right|}_{\text{A/m}} 2pa$
- For a thin short dipole ($\ell << l$) we assume that the current is constant and flows along the center of the wire; it is a filament of zero diameter. The two-dimensional integral over S becomes a one-dimensional integral over the length,

$$\iint_S \vec{J}_s ds' \to 2pa \int_L I \, d\vec{\ell}'$$

Hertzian Dipole (2)

Using $\vec{r}' = \hat{z}z'$ and $\hat{r} = \hat{x}\sin q \cos f + \hat{y}\sin q \sin f + \hat{z}\cos q$ gives $\vec{r}' \bullet \hat{r} = z'\cos q$. The radiation integral for the electric field becomes

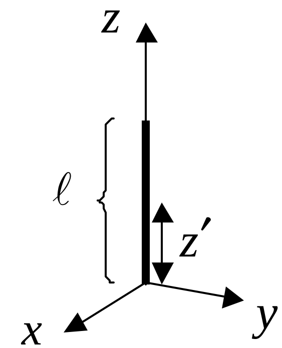
$$\vec{E}(r,q,f) \approx \frac{-jkh}{4pr} e^{-jkr} \int_0^{\ell} I\, e^{jk(\vec{r}'\bullet\hat{r})} \underbrace{\hat{z}dz'}_{d\vec{l}'} = \frac{-jkhI\,\hat{z}}{4pr} e^{-jkr} \int_0^{\ell} e^{jkz'\cos q}\, dz'$$

However, because ℓ is very short, $kz' \to 0$ and $e^{jkz'\cos q} \approx 1$. Therefore,

$$\vec{E}(r,q,f) \approx \frac{-jkhI\,\hat{z}}{4pr} e^{-jkr} \int_0^{\ell} (1)\, dz' = \frac{-jkhI\ell\,\hat{z}}{4pr} e^{-jkr}$$

leading to the spherical field components

$$E_q = \hat{q} \bullet \vec{E} \approx \frac{-jkhI\ell\,\hat{q} \bullet \hat{z}}{4pr} e^{-jkr}$$
$$= \frac{jkhI\ell \sin q}{4pr} e^{-jkr}$$
$$E_f = \hat{f} \bullet \vec{E} = 0$$



SHORT CURRENT FILAMENT

Hertzian Dipole (3)

Note that the electric field has only a 1/*r* dependence. The absence of higher order terms is due to the fact that the dipole is infinitesimal, and therefore $r_{ff} \to 0$. The field is a spherical wave and hence the TEM relationship can be used to find the magnetic field intensity

$$\vec{H} = \frac{\hat{k} \times \hat{E}}{h} = \frac{\hat{r} \times E_q \hat{q}}{h} = \hat{f}\, \frac{jkI\ell \sin q}{4pr} e^{-jkr}$$

The time-averaged Poynting vector is

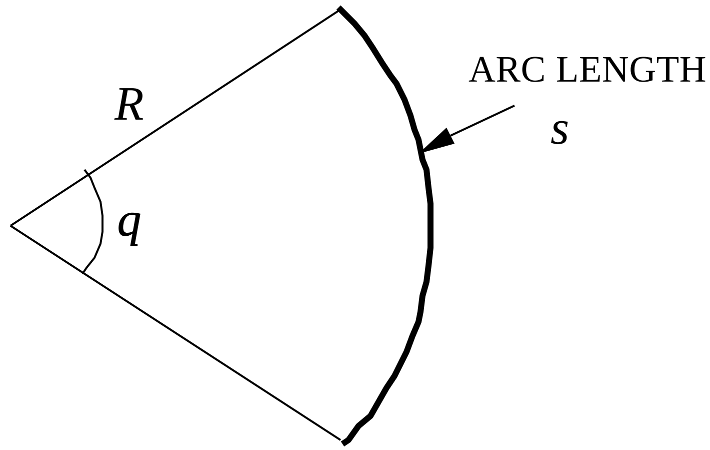
$$\vec{W}_{\text{av}} = \frac{1}{2}\Re\left\{\hat{E} \times \vec{H}^*\right\} = \frac{1}{2}\Re\left\{E_q H_f^*\right\}\hat{r} = \frac{hk^2 |I|^2 \ell^2 \sin^2 q}{32p^2 r^2}\hat{r}$$

The power flow is outward from the source, as expected for a spherical wave. The average power flowing through the surface of a sphere of radius *r* surrounding the source is

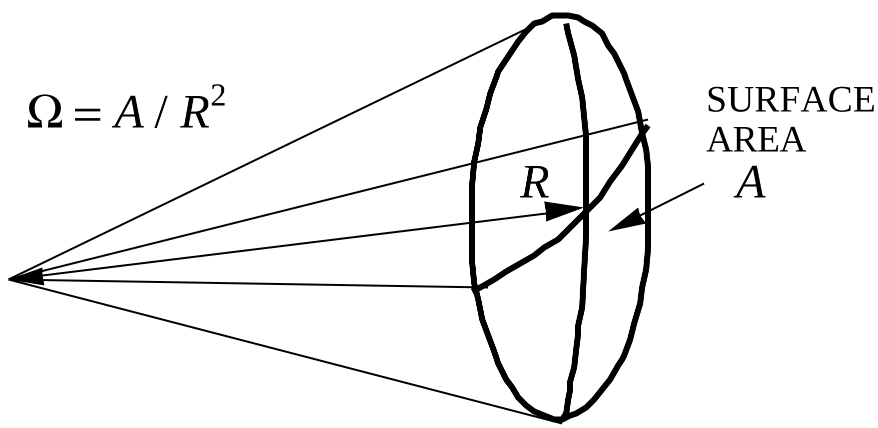
$$P_{\text{rad}} = \int_0^{2p}\int_0^{p} \vec{W}_{\text{av}} \bullet \hat{n}\, ds = \frac{hk^2 |I|^2 \ell^2}{32p^2 r^2} \underbrace{\int_0^{2p}\int_0^{p} \sin^2 q\; \hat{r} \bullet \hat{r}\, r^2 \sin q\, dq\, df}_{=8p/3} = \frac{hk^2 |I|^2 \ell^2}{12p} \text{ W}$$

Solid Angles and Steradians

Plane angles: $s = R\theta$, if $s = R$ then $\theta = 1$ radian



Solid angles: $\Omega = A / R^2$, if $A = R^2$, then $\Omega = 1$ steradian



Directivity and Gain (1)

The radiation intensity is defined as

$$U(q,f) = \frac{dP_{\text{rad}}}{d\Omega} = r^2 \hat{r} \bullet \vec{W}_{\text{av}} = r^2 \left|\vec{W}_{\text{av}}\right|$$

and has units of Watts/steradian (W/sr). The directivity function or directive gain is defined as

$$D(q,f) = \frac{\text{power radiated per unit solid angle}}{\text{average power radiated per unit solid angle}} = \frac{dP_{\text{rad}}\,/\,d\Omega}{P_{\text{rad}}\,/(4p)} = 4p\,\frac{r^2\left|\vec{W}_{\text{av}}\right|}{P_{\text{rad}}}$$

For the Hertzian dipole,

$$D(q,f) = 4p\,\frac{r^2\left|\vec{W}_{\text{av}}\right|}{P_{\text{rad}}} = 4p\,\frac{r^2\left|\dfrac{hk^2\left|I\right|^2\ell^2\sin^2 q}{32p^2 r^2}\right|}{\dfrac{hk^2\left|I\right|^2\ell^2}{12p}} = \frac{3}{2}\sin^2 q$$

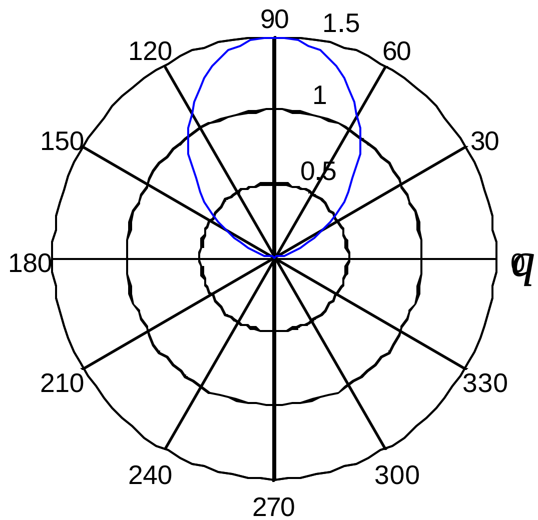
The directivity is the maximum value of the directive gain

$$D_o = D_{\max}(q,f) = D(q_{\max}, f_{\max}) = \frac{3}{2}$$

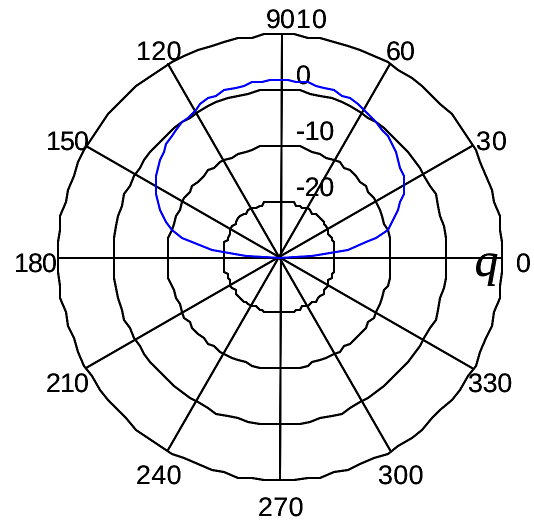
Dipole Polar Radiation Plots

Half of the radiation pattern of the dipole is plotted below for a fixed value of ϕ. The <u>half-power beamwidth</u> (HPBW) is the angular width between the half power points ($1/\sqrt{2}$ below the maximum on the voltage plot, or –3dB below the maximum on the decibel plot).

FIELD (VOLTAGE) PLOT



DECIBEL PLOT

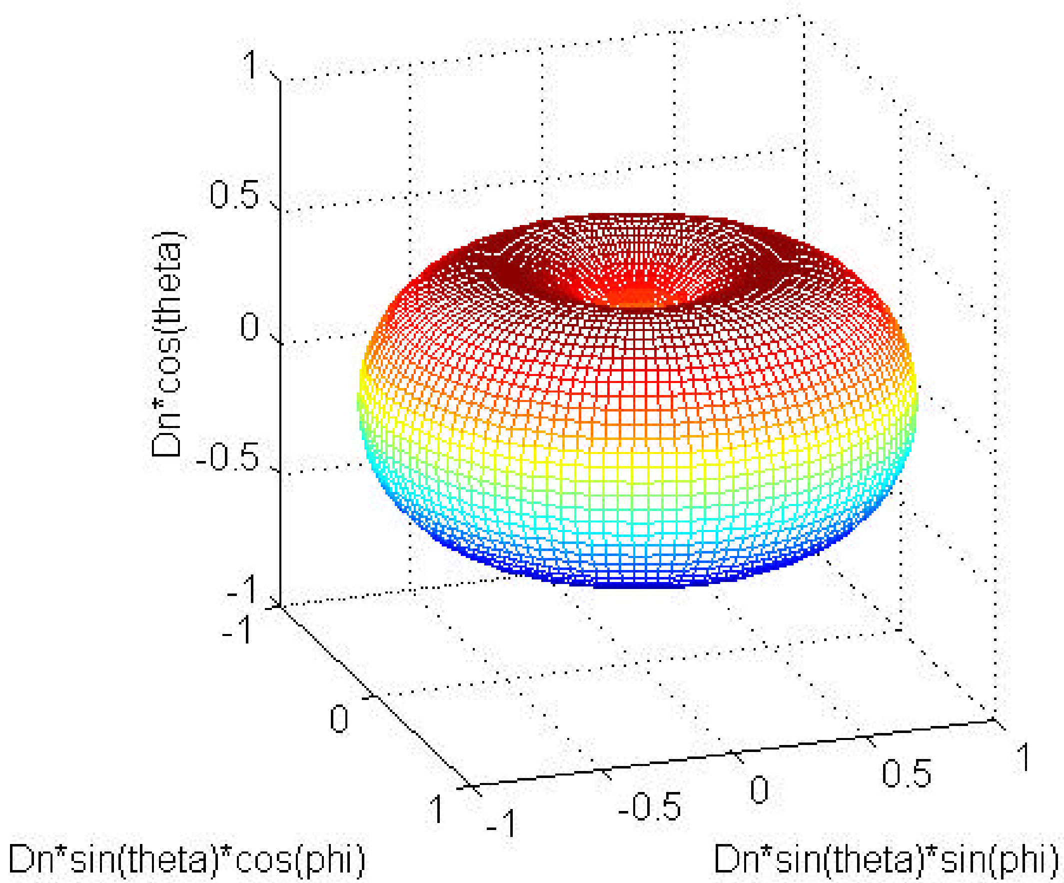


The half power beamwidth of the Hertzian dipole, θ_B:

$$|E_{\text{norm}}| \sim |\sin\theta| \quad \Rightarrow \quad \sin(\theta_{\text{HP}}) = 0.707 \quad \Rightarrow \quad \theta_{\text{HP}} = 45^\circ \quad \Rightarrow \quad \theta_B = 2\theta_{\text{HP}} = 90^\circ$$

Dipole Radiation Pattern

Radiation pattern of a Hertzian dipole aligned with the *z* axis. D_n is the normalized directivity. The directivity value is proportional to the distance from the center.



Directivity and Gain (2)

Another formula for directive gain is

$$D(q,f) = \frac{4p}{\Omega_A} \left| \vec{E}_{\mathrm{norm}}(q,f) \right|^2$$

where Ω_A is the <u>beam solid angle</u>

$$\Omega_A = \int_0^{2p} \int_0^{p} \left| \vec{E}_{\mathrm{norm}}(q,f) \right|^2 \sin q \, dq \, df$$

and $\left| \vec{E}_{\mathrm{norm}}(q,f) \right|$ is the normalized magnitude of the electric field pattern (i.e., the normalized radiation pattern)

$$\left| \vec{E}_{\mathrm{norm}}(q,f) \right| = \frac{\left| \vec{E}(r,q,f) \right|}{\left| \vec{E}_{\max}(r,q,f) \right|}$$

Note that both the numerator and denominator have the same $1/r$ dependence, and hence the ratio is independent of r. This approach is often more convenient because most of our calculations will be conducted directly with the electric field. Normalization removes all of the cumbersome constants.

Directivity and Gain (3)

As an illustration, we re-compute the directivity of a Hertzian dipole. Noting that the maximum magnitude of the electric field is occurs when $q = p/2$, the normalized electric field intensity is simply

$$\left|\vec{E}_{\text{norm}}(q,f)\right| = \left|\sin q\right|$$

The beam solid angle is

$$\Omega_A = \int_0^{2p}\int_0^{p} \left|\vec{E}_{\text{norm}}(q,f)\right|^2 \sin q \, dq \, df$$

$$= 2p \underbrace{\int_0^{p} \sin^3 q \, dq}_{=4/3} = \frac{8p}{3}$$

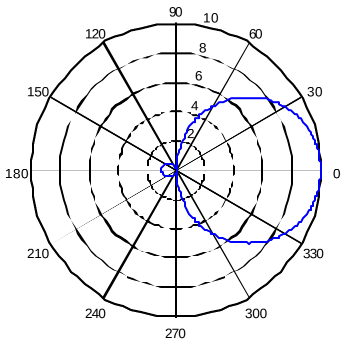
and from the definition of directivity,

$$D(q,f) = \frac{4p}{\Omega_A}\left|\vec{E}_{\text{norm}}(q,f)\right|^2 = \frac{4p}{8p/3}\left|\sin q\right|^2 = \frac{3}{2}\sin^2 q$$

which agrees with the previous result.

Example

Find the directivity of an antenna whose far-electric field is given by



$$E_\theta(r,\theta,\phi) = \begin{cases} \dfrac{10e^{-jkr}}{r}\cos\theta, & 0^\circ \le \theta \le 90^\circ \\ \dfrac{e^{-jkr}}{r}\cos\theta, & 90^\circ \le \theta \le 180^\circ \end{cases}$$

The maximum electric field occurs when $\cos\theta = 1 \quad \rightarrow \quad \left|\vec{E}_{\max}\right| = 10/r$. The normalized electric field intensity is

$$\left|E_{\theta_{\text{norm}}}(\theta,\phi)\right| = \begin{cases} |\cos\theta|, & 0^\circ \le \theta \le 90^\circ \\ 0.1|\cos\theta|, & 90^\circ \le \theta \le 180^\circ \end{cases}$$

which gives a beam solid angle of

$$\Omega_A = \int_0^{2\pi}\int_0^{\pi/2} \cos^2\theta \sin\theta \, d\theta \, d\phi + 0.01\int_0^{2\pi}\int_{\pi/2}^{\pi} \cos^2\theta \sin\theta \, d\theta \, d\phi = \frac{2\pi}{3}(1.1)$$

and a directivity of $D_o = 5.45 = 7.37$ dB.

Beam Solid Angle and Radiated Power

In the far field the radiated power is

$$P_{\text{rad}} = \frac{1}{2}\int_0^{2p}\int_0^{p} \underbrace{\Re\left\{\vec{E}\times\vec{H}^*\right\}}_{\left|\vec{E}\right|^2/h} \bullet \hat{r}\, ds = \frac{1}{2h}\underbrace{\int_0^{2p}\int_0^{p} \left|\vec{E}\right|^2 r^2 \sin q\, dq\, df}_{\equiv F_{\text{rad}}} \quad \Rightarrow \quad F_{\text{rad}} = 2hP_{\text{rad}}$$

From the definition of beam solid angle

$$\Omega_A = \int_0^{2p}\int_0^{p} \left|\vec{E}_{\text{norm}}(q,f)\right|^2 \sin q\, dq\, df$$

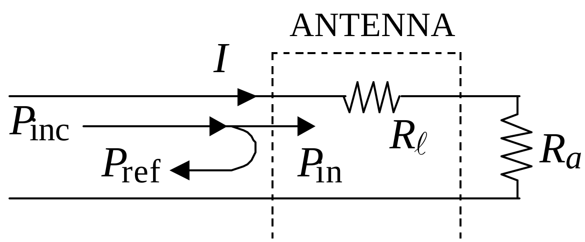
$$= \frac{1}{\left|\vec{E}_{\max}\right|^2 r^2}\underbrace{\int_0^{2p}\int_0^{p} \left|\vec{E}\right|^2 r^2 \sin q\, dq\, df}_{\equiv F_{\text{rad}}} = \quad \Rightarrow \quad F_{\text{rad}} = \Omega_A \left|\vec{E}_{\max}\right|^2 r^2$$

Equate the expressions for F_{rad}

$$P_{\text{rad}} = \frac{\Omega_A \left|\vec{E}_{\max}\right|^2 r^2}{2h}$$

Gain vs. Directivity (1)

Directivity is defined with respect to the radiated power, P_{rad}. This could be less than the power into the antenna if the antenna has losses. The gain is referenced to the power into the antenna, P_{in}.



Define the following:

P_{inc} = power incident on the antenna terminals
P_{ref} = power reflected at the antenna input
P_{in} = power into the antenna
P_{loss} = power loss in the antenna (dissipated in resistor R_ℓ, $P_{\mathrm{loss}} = \frac{1}{2}|I|^2 R_\ell$)
P_{rad} = power radiated (delivered to resistor R_a, $P_{\mathrm{rad}} = \frac{1}{2}|I|^2 R_a$, R_a is the <u>radiation resistance</u>)

The <u>antenna efficiency</u>, e, is $P_{\mathrm{rad}} = eP_{\mathrm{in}}$ where $0 \le e \le 1$.

Gain vs. Directivity (2)

Gain is defined as

$$G(\theta,\phi) = \frac{dP_{\rm rad}/d\Omega}{P_{\rm in}/(4\pi)} = 4\pi \frac{dP_{\rm rad}/d\Omega}{P_{\rm rad}/e} = eD(\theta,\phi)$$

Most often the use of the term gain refers to the maximum value of $G(\theta,\phi)$.

Example: The antenna input resistance is 50 ohms, of which 40 ohms is radiation resistance and 10 ohms is ohmic loss. The input current is 0.1 A and the directivity of the antenna is 2.

The input power is $P_{\rm in} = \frac{1}{2}|I|^2 R_{\rm in} = \frac{1}{2}|0.1|^2(50) = 0.25\ \text{W}$

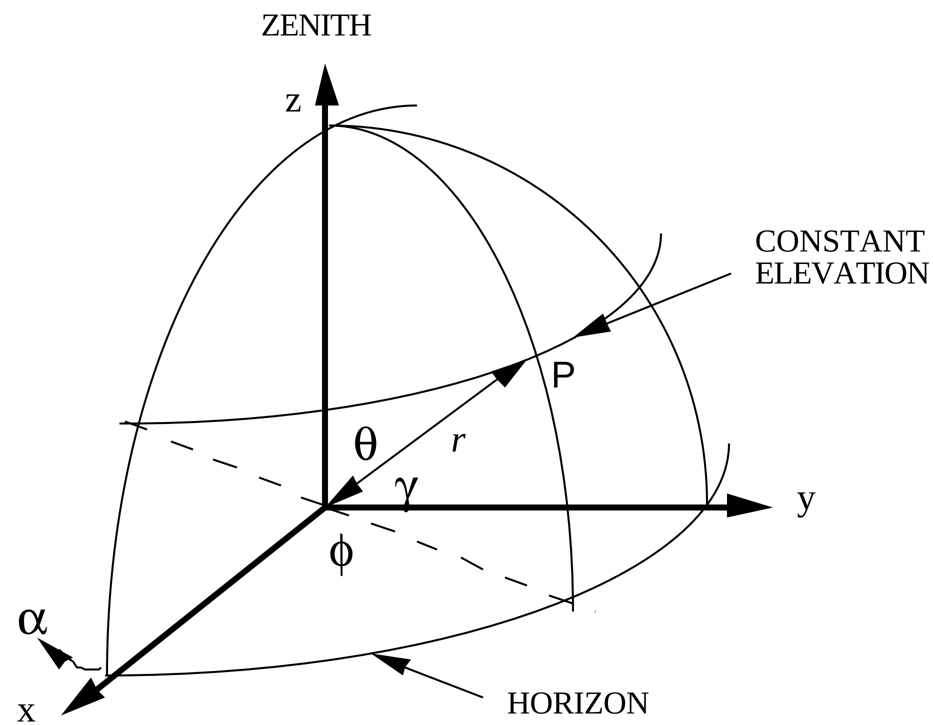
The power dissipated in the antenna is $P_{\rm loss} = \frac{1}{2}|I|^2 R_\ell = \frac{1}{2}|0.1|^2(10) = 0.05\ \text{W}$

The power radiated into space is $P_{\rm rad} = \frac{1}{2}|I|^2 R_a = \frac{1}{2}|0.1|^2(40) = 0.2\ \text{W}$

If the directivity is $D_o = 2$ then the gain is $G = eD = \left(\frac{P_{\rm rad}}{P_{\rm in}}\right)D = \left(\frac{0.2}{0.25}\right)(2) = 1.6$

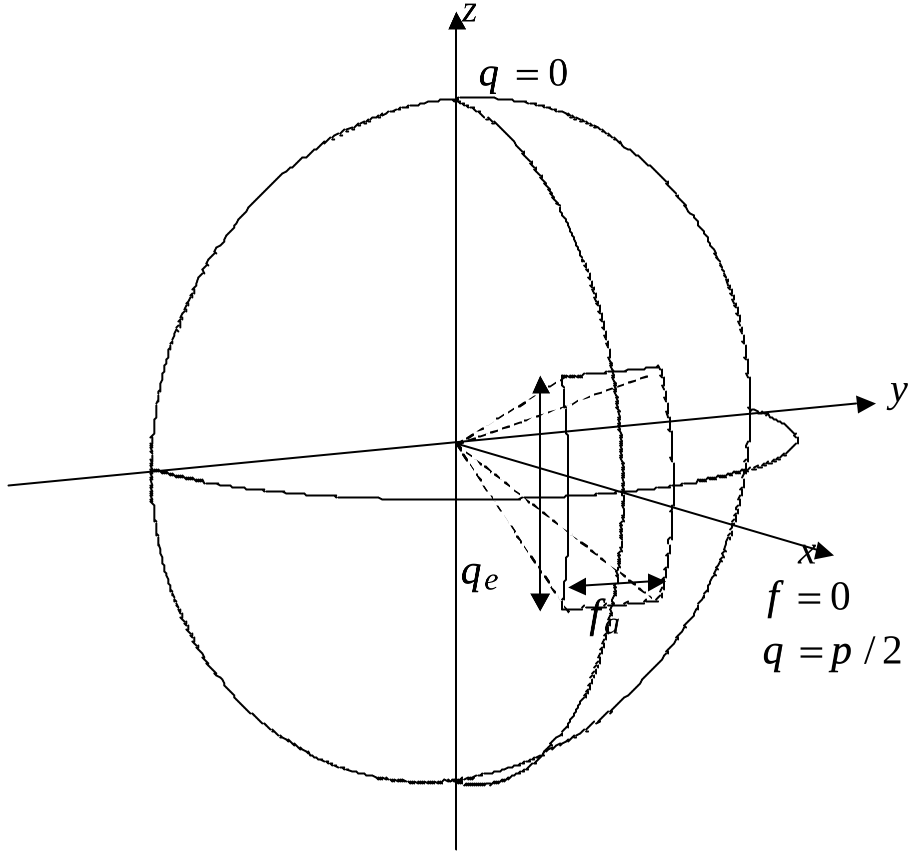
Azimuth/Elevation Coordinate System

Radars frequently use the azimuth/elevation coordinate system: (Az,El) or (a, g) or (q_e, f_a). The antenna is located at the origin of the coordinate system; the earth's surface lies in the *x*-*y* plane. Azimuth is generally measured clockwise from a reference (like a compass) but the spherical system azimuth angle f is measured counterclockwise from the *x* axis. Therefore $a = 360 - f$ and $g = 90 - q$ degrees.



Approximate Directivity Formula (1)

Assume the antenna radiation pattern is a “pencil beam” on the horizon. The pattern is constant inside of the elevation and azimuth half power beamwidths (q_e, f_a) respectively:



Approximate Directivity Formula (2)

Approximate antenna pattern

$$\vec{E}(\theta,\phi) = \begin{cases} \dfrac{E_o e^{-jkr}}{r}\hat{\theta}, & (\pi/2-\theta_e/2) \le \theta \le (\pi/2+\theta_e/2) \text{ and } -\phi_a/2 \le \phi \le \phi_a/2 \\ 0, & \text{else} \end{cases}$$

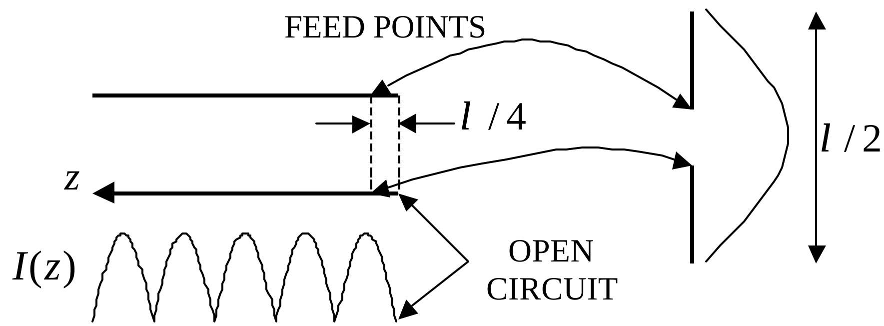
The beam solid angle is

$$\begin{aligned} \Omega_A &= \int_{\frac{\pi}{2}-\frac{\theta_e}{2}}^{\frac{\pi}{2}+\frac{\theta_e}{2}} \int_{-\frac{\phi_a}{2}}^{\frac{\phi_a}{2}} \underbrace{\sin\theta}_{\approx 1}\, d\theta\, d\phi \\ &= \phi_a\left[\sin(\theta_e/2) - \sin(-\theta_e/2)\right] \\ &\approx \phi_a\left[\theta_e/2 - (-\theta_e/2)\right] = \phi_a\theta_e \end{aligned}$$

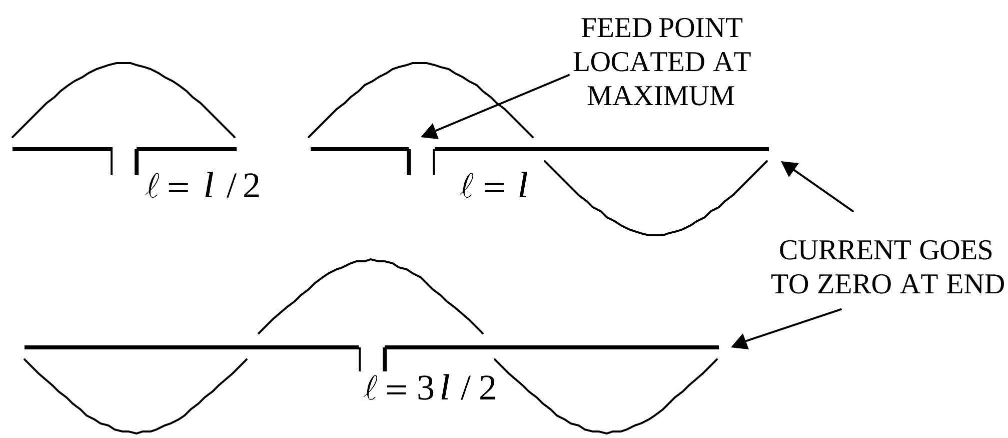
This leads to an approximation for the directivity of $D_o = \dfrac{4\pi}{\Omega_A} = \dfrac{4\pi}{\theta_e\phi_a}$. Note that the angles are in radians. This formula is often used to estimate the directivity of an omni-directional antenna with negligible sidelobes.

Thin Wire Antennas (1)

Thin wire antennas satisfy the condition $a << l$. If the length of the wire (ℓ) is an integer multiple of a half wavelength, we can make an "educated guess" at the current based on an open circuited two-wire transmission line



For other multiples of a half wavelength the current distribution has the following features



Thin Wire Antennas (2)

On a half-wave dipole the current can be approximated by

$$I(z) = I_o \cos(kz) \quad \text{for} - l/4 < z < l/4$$

Using this current in the radiation integral

$$\vec{E}(r,q,f) = \frac{-jkh}{4pr} e^{-jkr} \hat{z} \int_{-l/4}^{l/4} I_o \cos(kz') e^{jkz' \cos q} dz'$$

$$= \frac{-jkhI_o}{4pr} e^{-jkr} \hat{z} \int_{-l/4}^{l/4} \cos(kz') e^{jkz' \cos q} dz'$$

From a table of integrals we find that

$$\int \cos(Bz') e^{Az'} dz' = \frac{e^{Az'} [A \overbrace{\cos(Bz')}^{=0} + B \overbrace{\sin(Bz')}^{=\pm 1}]}{A^2 + B^2}$$

where $A = jk \cos q$ and $B = k$, so that $A^2 + B^2 = -k^2 \cos^2 q + k^2 = k^2 \sin^2 q$. The q component requires the dot product $\hat{z} \bullet \hat{q} = -\sin q$.

Thin Wire Antennas (3)

Evaluating the limits gives

$$E_q = \frac{jkhI_o}{4pr} e^{-jkr} \sin q \, \frac{k\left[\overbrace{e^{jp\cos q/2} - (-1)e^{-jp\cos q/2}}^{2\cos\left(\frac{p}{2}\cos q\right)}\right]}{k^2 \sin^2 q} = \frac{jhI_o}{2pr} e^{-jkr} \frac{\cos\left(\frac{p}{2}\cos q\right)}{\sin q}$$

The magnetic field intensity in the far field is

$$\vec{H} = \frac{\hat{k}\times\vec{E}}{h} = \hat{f}\frac{E_q}{H_f} = \frac{jI_o}{2pr} e^{-jkr} \frac{\cos\left(\frac{p}{2}\cos q\right)}{\sin q}\hat{f}$$

The directivity is computed from the beam solid angle, which requires the normalized electric field intensity

$$\left|\vec{E}_{\text{norm}}\right|^2 = \frac{\left|E_q\right|^2}{\left|E_{q\,\max}\right|^2} = \left|\frac{\cos\left(\frac{p}{2}\cos q\right)}{\sin q}\right|^2$$

Thin Wire Antennas (4)

$$\Omega_A = 2p \int_0^p \frac{\cos^2(p \cos q / 2)}{\sin^2 q} \sin q \, dq = 2p \underbrace{\int_0^p \frac{\cos^2(p \cos q / 2)}{\sin^2 q} \, dq}_{\text{Integrate numerically}} = (2p)(1.218)$$

The directive gain is

$$D = \frac{4p}{\Omega_A} \left| \vec{E}_{\text{norm}}(q, f) \right|^2 = \frac{4p}{(2p)(1.218)} \frac{\cos^2\left(\frac{p}{2}\cos q\right)}{\sin^2 q} = 1.64 \frac{\cos^2\left(\frac{p}{2}\cos q\right)}{\sin^2 q}$$

The radiated power is

$$P_{\text{rad}} = \frac{\Omega_A \left| E_{q\,\max} \right|^2 r^2}{2h} = \frac{2p(1.218)h^2 |I_o|^2 r^2}{2h(2pr)^2} = 36.57 |I_o|^2 \equiv \frac{1}{2} |I_o|^2 R_a$$

where R_a is the <u>radiation resistance</u> of the dipole. The radiated power can be viewed as the power delivered to resistor that represents "free space." For the half-wave dipole the radiation resistance is

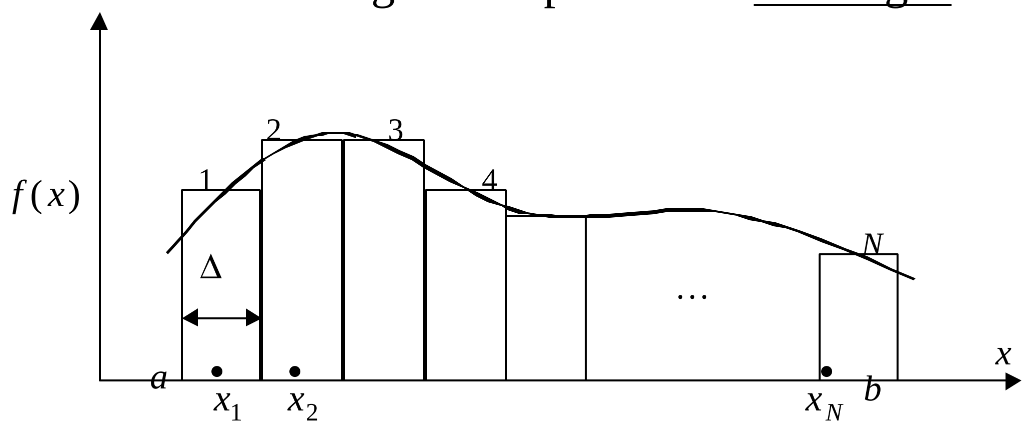
$$R_a = \frac{2P_{\text{rad}}}{|I_o|^2} = (2)(36.57) = 73.13 \text{ ohms}$$

Numerical Integration (1)

The rectangular rule is a simple way of evaluating an integral numerically. The area under the curve of $f(x)$ is approximated by a sum of rectangular areas of width Δ and height $f(x_n)$, where $x_n = \frac{\Delta}{2} + (n-1) + a$ is the center of the interval *n*th interval. Therefore, if all of the rectangles are of equal width

$$\int_a^b f(x)\,dx \approx \Delta \sum_a^b f(x_n)$$

Clearly the approximation can be made as close to the exact value as desired by reducing the width of the triangles as necessary. However, to keep computation time to a minimum, only the smallest number of rectangles that provides a converged solution should be used.



Numerical Integration (2)

Example: Matlab programs to integrate $\int_0^{\pi} \frac{\cos^2(\pi \cos\theta / 2)}{\sin\theta} d\theta$

Sample Matlab code for the rectangular rule

```
% integrate dipole pattern using the rectangular rule
clear
rad=pi/180;
% avoid 0 by changing the limits slightly
a=.001; b=pi-.001;
N=5
delta=(b-a)/N;
sum=0;
for n=1:N
      theta=delta/2+(n-1)*delta;
      sum=sum+cos(pi*cos(theta)/2)^2/sin(theta);
end
I=sum*delta
```

Convergence: N=5, 1.2175; N=10, 1.2187; N=50, 1.2188. Sample Matlab code using the quad8 function

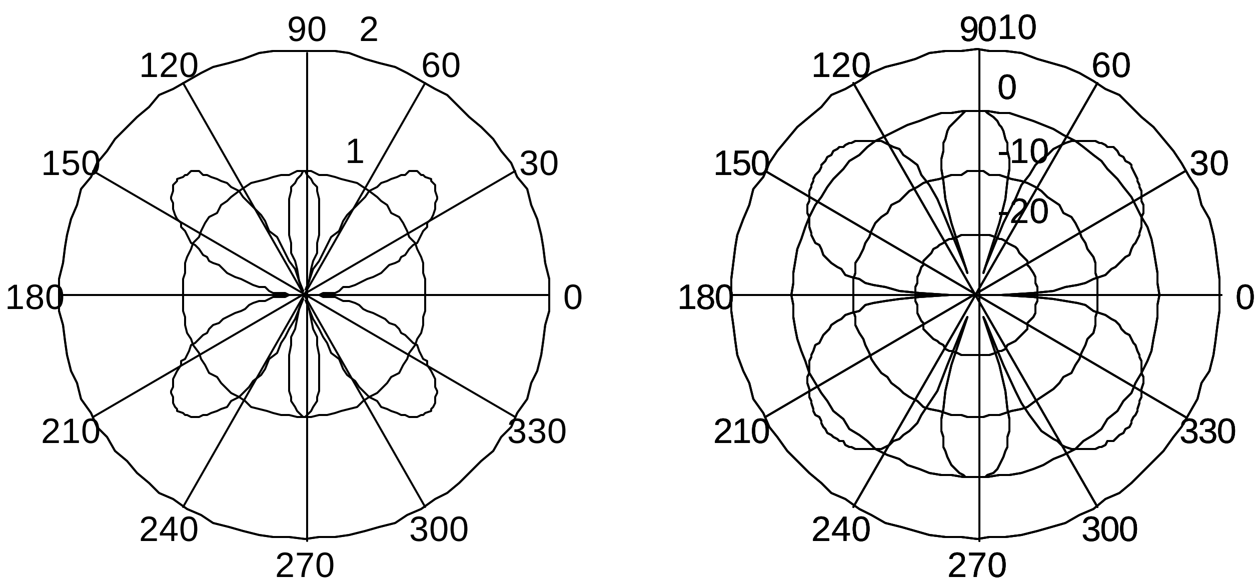
```
% integrate to find half wave dipole solid angle
clear
I=quad8('cint',0.0001,pi-.0001,.00001);
disp(['cint integral, I: ',num2str(I)])

function P=cint(T)
% function to be integrated
P=(cos(pi*cos(T)/2).^2)./sin(T);
```

Thin Wires of Arbitrary Length

For a thin-wire antenna of length ℓ along the z axis, the electric field intensity is

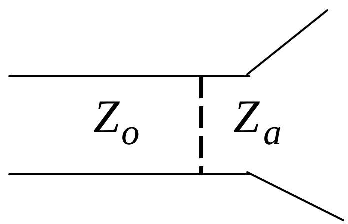
$$E_\theta = \frac{j\eta I}{2\pi r} e^{-jkr} \left[\frac{\cos\left(\frac{k\ell}{2}\cos\theta\right) - \cos\left(\frac{k\ell}{2}\right)}{\sin\theta} \right]$$

Example: $\ell = 1.5\lambda$ (left: voltage plot; right: decibel plot)



Feeding and Tuning Wire Antennas (1)

When an antenna terminates a transmission line, as shown below, the antenna impedance (Z_a) should be matched to the transmission line impedance (Z_o) to maximize the power delivered to the antenna



$$\Gamma = \frac{Z_a - Z_o}{Z_a + Z_o} \quad \text{and} \quad \text{VSWR} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

The antenna's input impedance is generally a complex quantity, $Z_a = (R_a + R_\ell) + jX_a$. The approach for matching the antenna and increasing its efficiency is

1. minimize the ohmic loss, $R_\ell \rightarrow 0$
2. "tune out" the reactance by adjusting the antenna geometry or adding lumped elements, $X_a \rightarrow 0$ (<u>resonance</u> occurs when Z_a is real)
3. match the radiation resistance to the characteristic impedance of the line by adjusting the antenna parameters or using a transformer section, $R_a \rightarrow Z_o$

Feeding and Tuning Wire Antennas (2)

Example: A half-wave dipole is fed by a 50 ohm line

$$\Gamma = \frac{Z_a - Z_o}{Z_a + Z_o} = \frac{73 - 50}{73 + 50} = 0.1870$$

$$\text{VSWR} = \frac{1 + |\Gamma|}{1 - |\Gamma|} = 1.46$$

The loss due to reflection at the antenna terminals is

$$|t|^2 = 1 - |\Gamma|^2 = 0.965$$

$$10 \log\left(|t|^2\right) = -0.155 \text{ dB}$$

which is stated as "0.155 dB of reflection loss" (the negative sign is implied by using the word "loss").

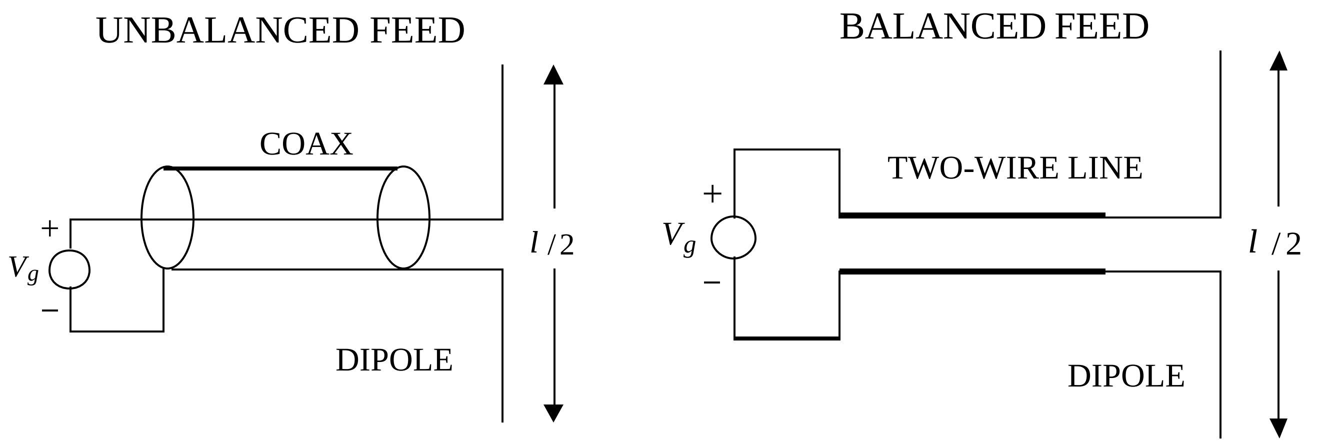
Feeding and Tuning Wire Antennas (3)

The antenna impedance is affected by

1. length
2. thickness
3. shape
4. feed point (location and method of feeding)
5. end loading

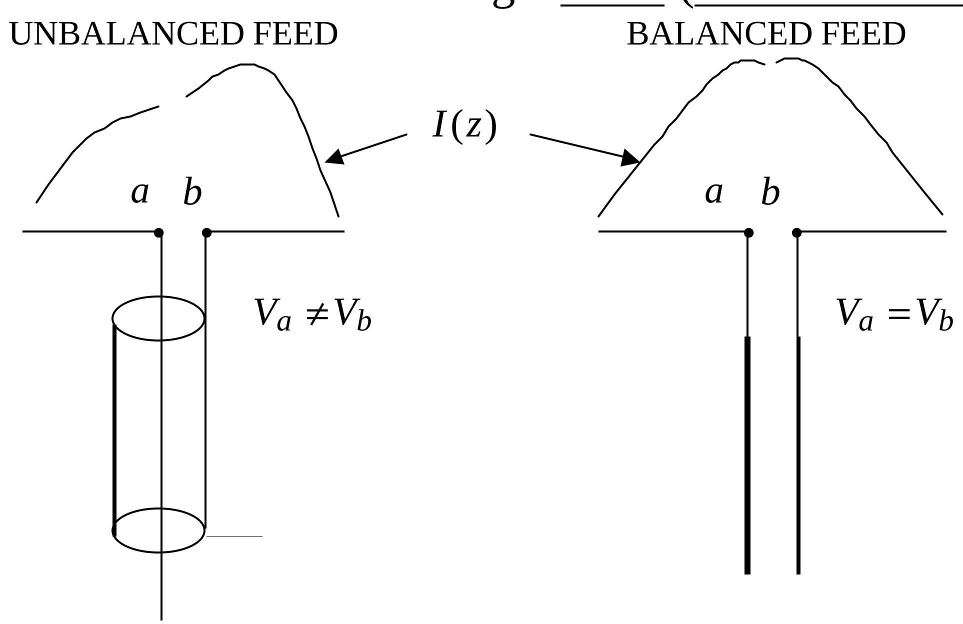
Although all of these parameters affect both the real and imaginary parts of Z_a, they are generally used to remove the reactive part. The remaining real part can be matched using a transformer section.

Another problem is encountered when matching a balanced radiating structure like a dipole to an unbalanced transmission line structure like a coax.



Feeding and Tuning Wire Antennas (4)

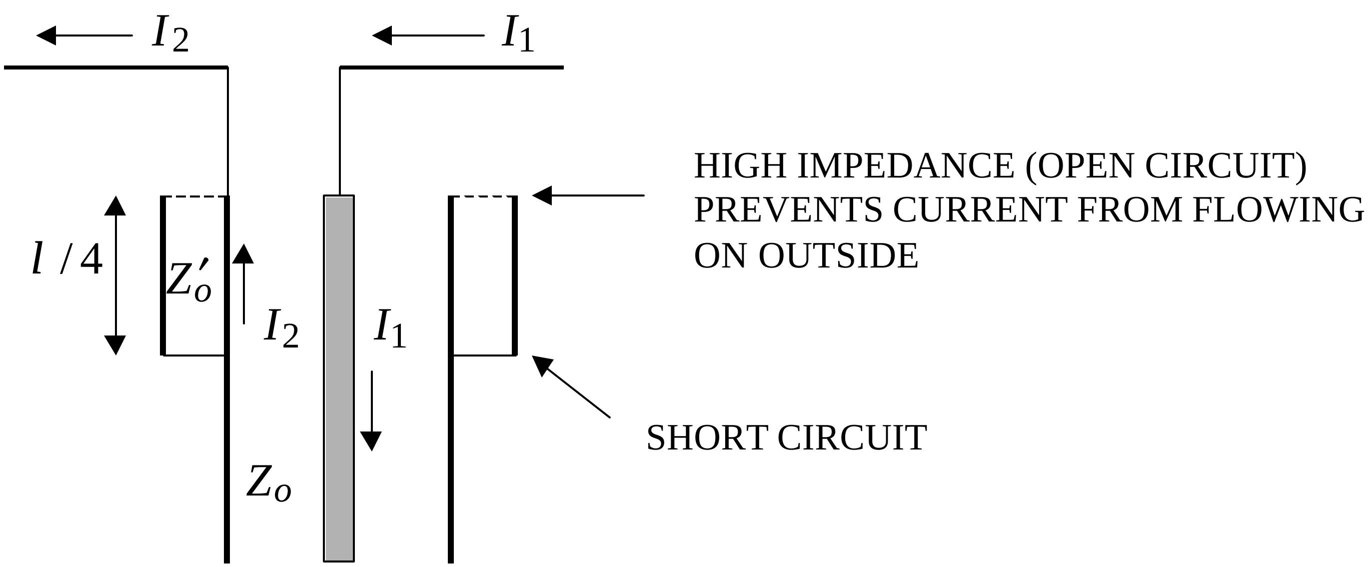
If the two structures are not balanced, a return current can flow on the outside of the coaxial cable. These currents will radiate and modify the pattern of the antenna. The unbalanced currents can be eliminated using a balun (balanced-to-unbalanced transformer)



Baluns frequently incorporate chokes, which are circuits designed to "choke off" current by presenting an open circuit to current waves.

Feeding and Tuning Wire Antennas (5)

An example of a balun employing a choke is the sleeve or bazooka balun

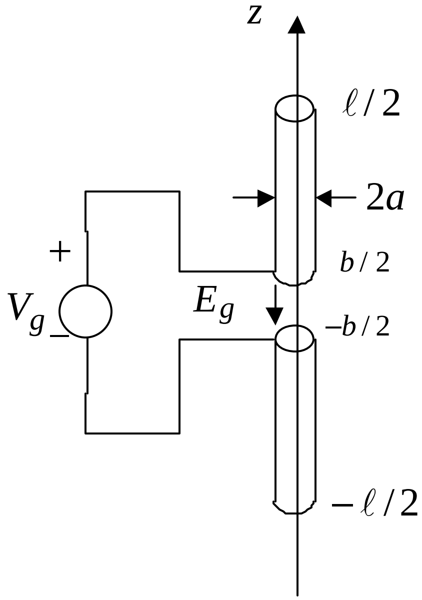


The choke prevents current from flowing on the exterior of the coax. All current is confined to the inside of surfaces of the coax, and therefore the current flow in the two directions is equal (balanced) and does not radiate. The integrity of a short circuit is easier to control than that of an open circuit, thus short circuits are used whenever possible. Originally balun referred exclusively to these types of wire feeding circuits, but the term has evolved to refer to any feed point matching circuit.

Calculation of Antenna Impedance (1)

The antenna impedance must be matched to that of the feed line. The impedance of an antenna can be measured or computed. Usually measurements are more time consuming (and therefore expensive) relative to computer simulations. However, for a simulation to accurately include the effect of all of the antenna's geometrical and electrical parameters on Z_a, a fairly complicated analytical model must be used. The resulting equations must be solved numerically in most cases.

One popular technique is the <u>method of moments</u> (MM) solution of an <u>integral equation</u> (IE) for the current.



1. $I(z')$ is the unknown current distribution on the wire
2. Find the z component of the electric field in terms of $I(z')$ from the radiation integral
3. Apply the boundary condition

$$E_z(r=a) = \begin{cases} 0, & b/2 \le |z| \le \ell/2 \\ E_g, & b/2 \ge |z| \end{cases}$$

in order to obtain an integral equation for $I(z')$

Calculation of Antenna Impedance (2)

One special form of the integral equation for thin wires is Pocklington's equation

$$\left(k^2+\frac{\partial^2}{\partial z^2}\right)\int_{-\ell/2}^{\ell/2} I(z')\frac{e^{-jkR}}{4pR}dz' = \begin{cases} 0, & b/2 \le |z| \le \ell/2 \\ -jweE_g, & b/2 \ge |z| \end{cases}$$

where $R=\sqrt{a^2+(z-z')^2}$. This is called an integral equation because the unknown quantity $I(z')$ appears in the integrand.

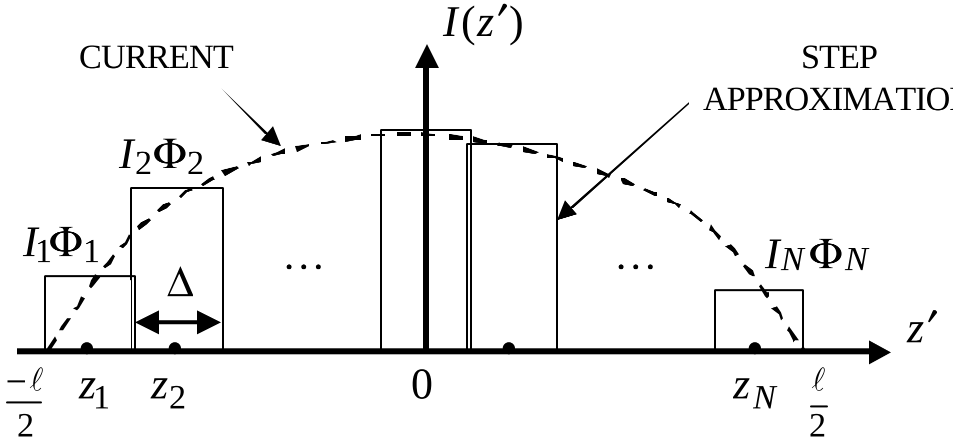
4. Solve the integral equation using the method of moments (MM). First approximate the current by a series with unknown expansion coefficients $\{I_n\}$

$$I(z')=\sum_{n=1}^{N} I_n \Phi_n(z')$$

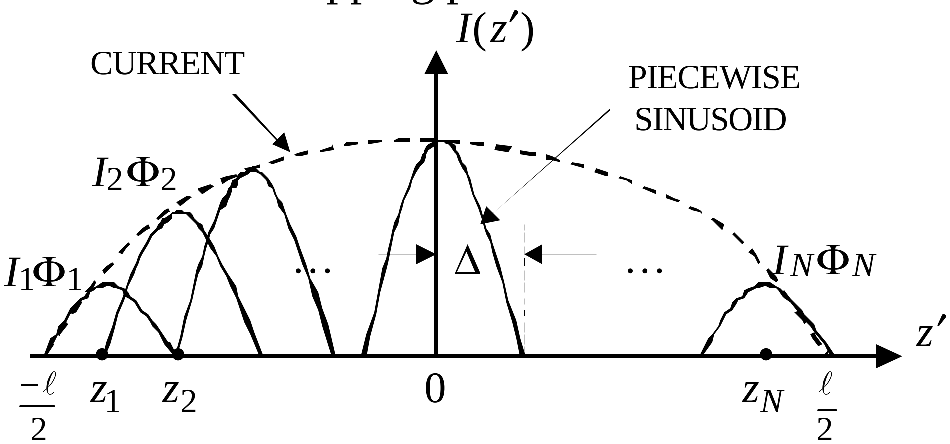
The basis functions or expansion functions $\{\Phi_n\}$ are known and selected to suit the particular problem. We would like to use as few basis functions as possible for computational efficiency, yet enough must be used to insure convergence.

Calculation of Antenna Impedance (3)

Example: a step approximation to the current using a series of pulses. Each segment is called a <u>subdomain</u>. Problem: there will be discontinuities between steps.

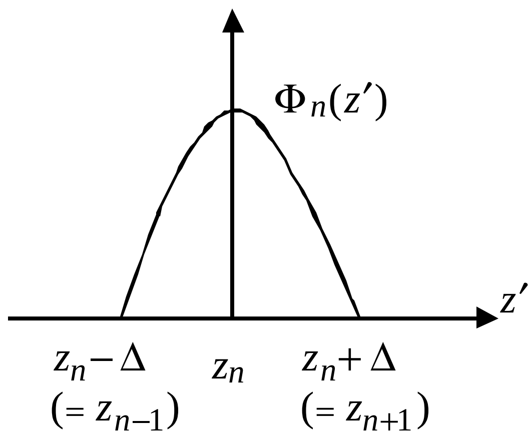


A better basis function is the overlapping piecewise sinusoid



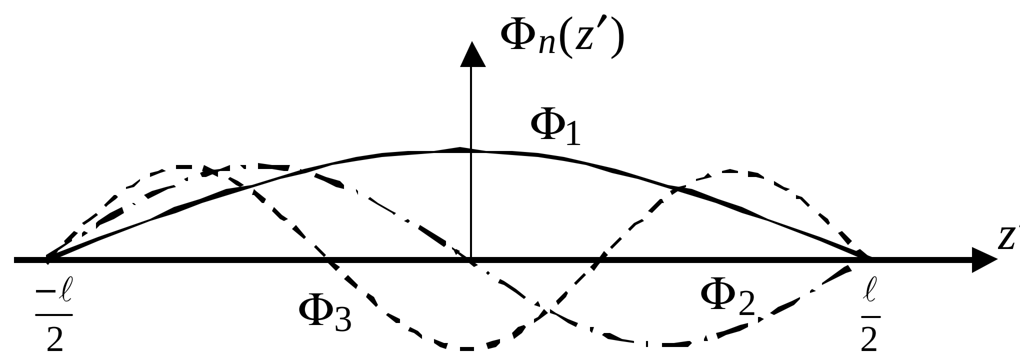
Calculation of Antenna Impedance (4)

A piecewise sinusoid extends over two segments (each of length Δ) and has a maximum at the point between the two segment



$$\Phi_n(z') = \begin{cases} \dfrac{z' - z'_{n-1}}{z'_n - z'_{n-1}}, & z'_{n-1} \le z' \le z'_n \\ \dfrac{z'_{n+1} - z'}{z'_{n+1} - z'_n}, & z'_n \le z' \le z'_{n+1} \\ 0, & \text{elsewhere} \end{cases}$$

Entire domain functions are also possible. Each entire domain basis function extends over the entire wire. Examples are sinusoids.



Calculation of Antenna Impedance (5)

Solving the integral equation: (1) insert the series back into the integral equation

$$\left(k^2+\frac{\partial^2}{\partial z^2}\right)\int_{-\ell/2}^{\ell/2}\left(\sum_{n=1}^{N} I_n\Phi_n(z')\right)\frac{e^{-jkR}}{4pR}dz' = \begin{cases}0, & b/2 \le |z| \le \ell/2\\ -jweE_g, & b/2 \ge |z|\end{cases}$$

Note that the derivative is with respect to z (not z') and therefore the differential operates only on R. For convenience we define new functions f and g:

$$\sum_{n=1}^{N} I_n \underbrace{\left\{\int_{-\ell/2}^{\ell/2}\left(k^2+\frac{\partial^2}{\partial z^2}\right)\Phi_n(z')\frac{e^{-jkR(z,z')}}{4pR(z,z')}dz'\right\}}_{\equiv f(\Phi_n)\to f_n(z)} = \underbrace{\begin{cases}0, & b/2 \le |z| \le \ell/2\\ -jweE_g, & b/2 \ge |z|\end{cases}}_{\equiv g}$$

Once Φ_n is defined, the integral can be evaluated numerically. The result will still be a function of z, hence the notation $f_n(z)$.

(2) Choose a set of N <u>testing (or weighting) functions</u> $\{\mathrm{X}_m\}$. Multiply both sides of the equation by each testing function and integrate over the domain of each function Δ_m to obtain N equations of the form

$$\int_{\Delta_m}\mathrm{X}_m(z)\sum_{n=1}^{N} I_n f_n(z)\,dz = \int_{\Delta_m}\mathrm{X}_m(z)g(z)\,dz, \quad m=1,2,\ldots,N$$

Calculation of Antenna Impedance (6)

Interchange the summation and integration operations and define new impedance and voltage quantities

$$\sum_{n=1}^{N} I_n \underbrace{\left\{ \int_{\Delta_m} \mathrm{X}_m(z) f_{mn}(z) dz \right\}}_{Z_{mn}} = \underbrace{\int_{\Delta_m} \mathrm{X}_m(z) g(z) dz}_{V_m}$$

This can be cast into the form of a matrix equation and solved using standard matrix methods

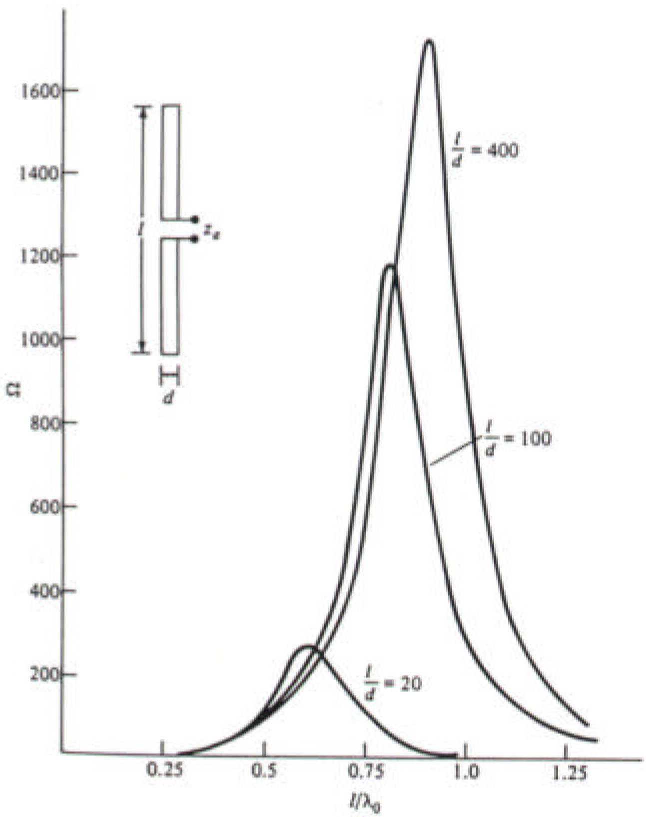
$$\sum_{n=1}^{N} I_n Z_{mn} = V_m \quad \rightarrow \quad [Z][I] = [V] \quad \rightarrow \quad [I] = [Z]^{-1}[V]$$

$[Z]$ is a square <u>impedance matrix</u> that depends only on the geometry and material characteristics of the dipole. Physically, it is a measure of the interaction between the currents on segments m and n. $[V]$ is the <u>excitation vector</u>. It depends on the field in the gap and the chosen basis functions. $[I]$ is the unknown current coefficient vector. After $[I]$ has been determined, the resulting current series can be inserted in the radiation integral, and the far fields computed by integration of the current.

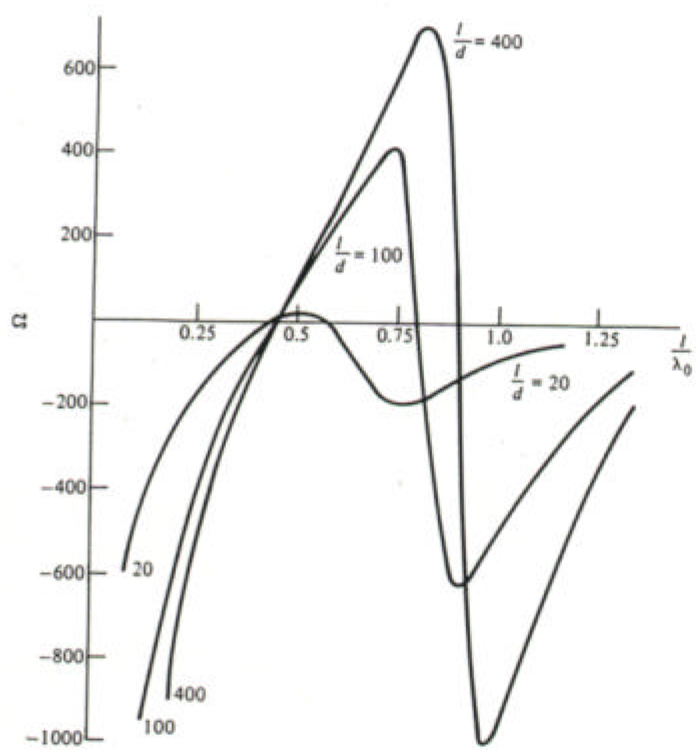
Self-Impedance of a Wire Antenna

The method of moments current allows calculation of the self impedance of the antenna by taking the ratio $Z_{\text{self}} = V_g / I_o$

RESISTANCE



REACTANCE



The Fourier Series Analog to MM

The method of moments is a general solution method that is widely used in all of engineering. A Fourier series approximation to a periodic time function has the same solution process as the MM solution for current. Let $f(t)$ be the time waveform

$$f(t) = \frac{a_o}{T} + \frac{2}{T}\sum_{n=1}^{\infty}\left[a_n \cos(w_n t) + b_n \sin(w_n t)\right]$$

For simplicity, assume that there is no DC component and that only cosines are necessary to represent $f(t)$ (true if the waveform has the right symmetry characteristics)

$$f(t) = \frac{2}{T}\sum_{n=1}^{\infty} a_n \cos(w_n t)$$

The constants are obtained by multiplying each side by the testing function $\cos(w_m t)$ and integrating over a period

$$\int_{-T/2}^{T/2} f(t)\cos(w_m t)dt = \frac{2}{T}\int_{-T/2}^{T/2}\left(\sum_{n=1}^{\infty} a_n \cos(w_n t)\right)\cos(w_m t)\,dt = \begin{cases} 0, & m \neq n \\ a_n, & m = n \end{cases}$$

This is analogous to MM when $f(t) \to I(z')$, $a_n \to I_n$, $\Phi_n \to \cos(w_n t)$, and $\mathrm{X}_m \to \cos(w_m t)$. (Since $f(t)$ is not in an integral equation, a second variable t' is not required.) The selection of the testing functions to be the complex conjugates of the expansion functions is referred to as Galerkin's method.

Reciprocity (1)

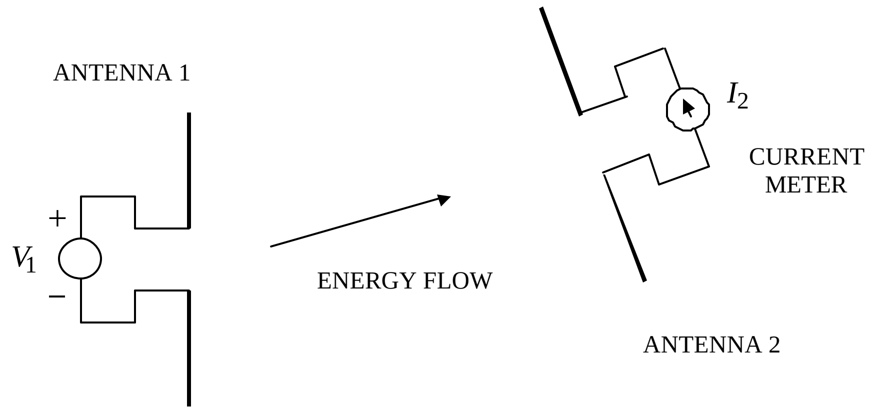
When two antennas are in close proximity to each other, there is a strong interaction between them. The radiation from one affects the current distribution of the other, which in turn modifies the current distribution of the first one.

Consider two situations (depicted on the following page) where the geometrical relationship between two antennas does not change.

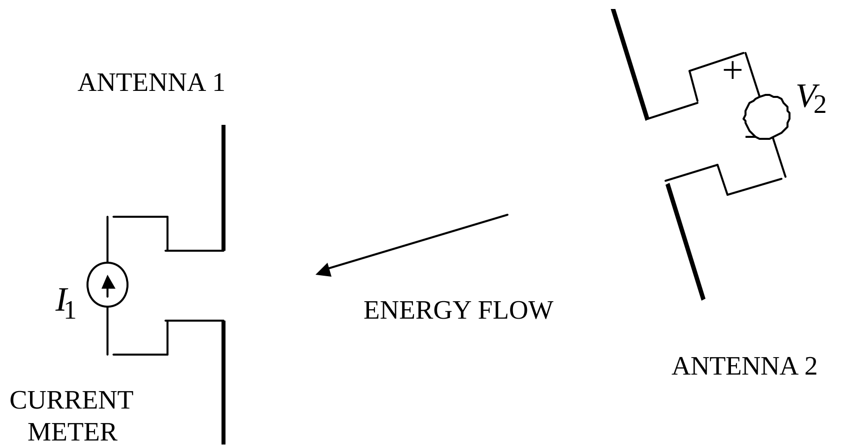
1. A voltage is applied to antenna 1 and the current induced at the terminals of antenna 2 is measured.
2. The situation is reversed: a voltage is applied to antenna 2 and the current induced at the terminals of antenna 1 is measured.

Reciprocity (2)

Case 1:



Case 2:



Mutual Impedance (1)

Define the mutual impedance or transfer impedance as

$$Z_{12} = \frac{V_2}{I_1} \quad \text{and} \quad Z_{21} = \frac{V_1}{I_2}$$

The first index on Z refers to the receiving antenna (observer) and the second index to the source antenna.

Reciprocity Theorem: If the antennas and medium are linear, passive and isotropic, then the response of a system to a source is unchanged if the source and observer (measurer) are interchanged.

With regard to mutual impedance:

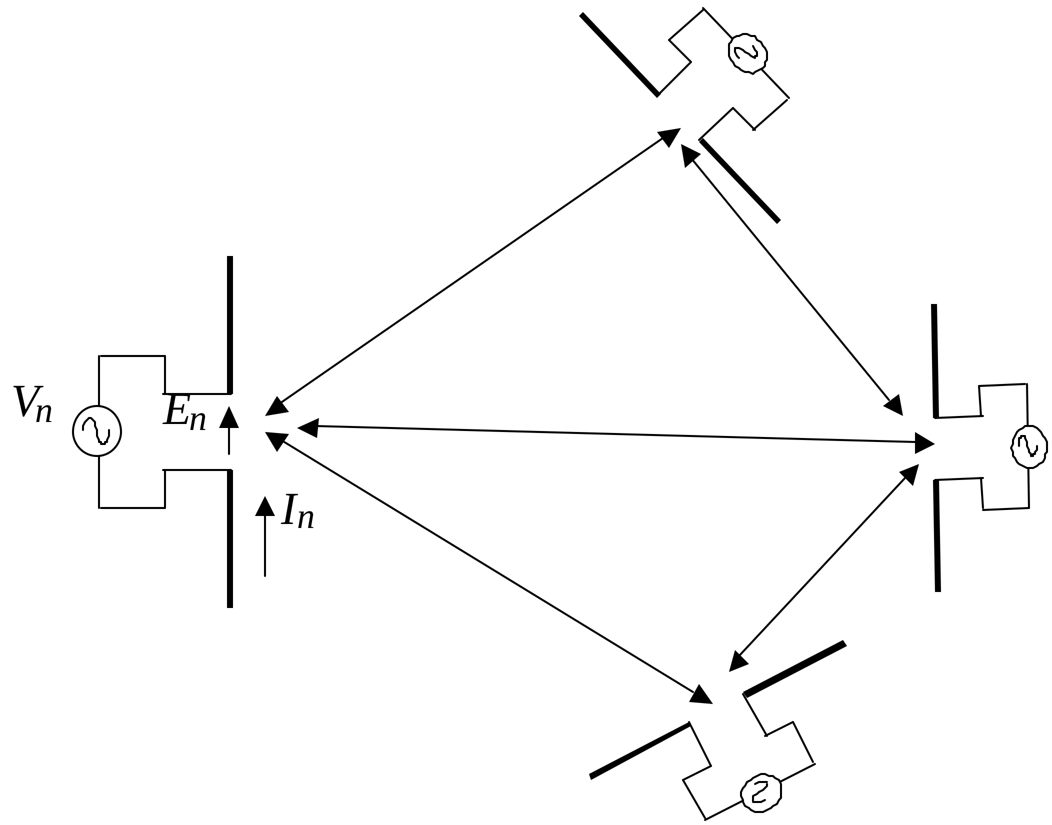
- This implies that $Z_{21} = Z_{12}$
- The receiving and transmitting patterns of an antenna are the same if it is constructed of linear, passive, and isotropic materials and devices.

In general, the input impedance of antenna number n in the presence of other antennas is obtained by integrating the total field in the gap (gap width b_n)

$$Z_n = \frac{V_n}{I_n} = \frac{1}{I_n} \int_{b_n} \vec{E}_n \bullet d\vec{\ell}$$

Mutual Impedance (2)

$\vec{E}_n$ is the total field in the gap of antenna n (due to its own voltage plus the incident fields from all other antennas).



Mutual Impedance (3)

If there are a total of N antennas

$$\vec{E}_n = \vec{E}_{n1} + \vec{E}_{n2} + \cdots + \vec{E}_{nN}$$

Therefore,

$$Z_n = \frac{1}{I_n} \int_{b_n} \sum_{m=1}^{N} \vec{E}_{nm} \bullet d\vec{\ell}_n$$

Define

$$V_{nm} = \int_{b_n} \vec{E}_{nm} \bullet d\vec{\ell}_n$$

The impedance becomes

$$Z_n = \frac{1}{I_n} \underbrace{\sum_{m=1}^{N} V_{nm}}_{\equiv V_n} = \sum_{m=1}^{N} \underbrace{\frac{V_{nm}}{I_n}}_{\equiv Z_{nm}} = \sum_{m=1}^{N} Z_{nm}$$

For example, the impedance of dipole n=1 is written explicitly as

$$Z_1 = Z_{11} + Z_{12} + \cdots + Z_{1N}$$

When $m = n$ the impedance is the <u>self impedance</u>. This is approximately the impedance that we have already computed for an isolated dipole using the method of moments.

Mutual Impedance (4)

The method of moments can also be used to compute the mutual coupling between antennas. The mutual impedance is obtained from the definition

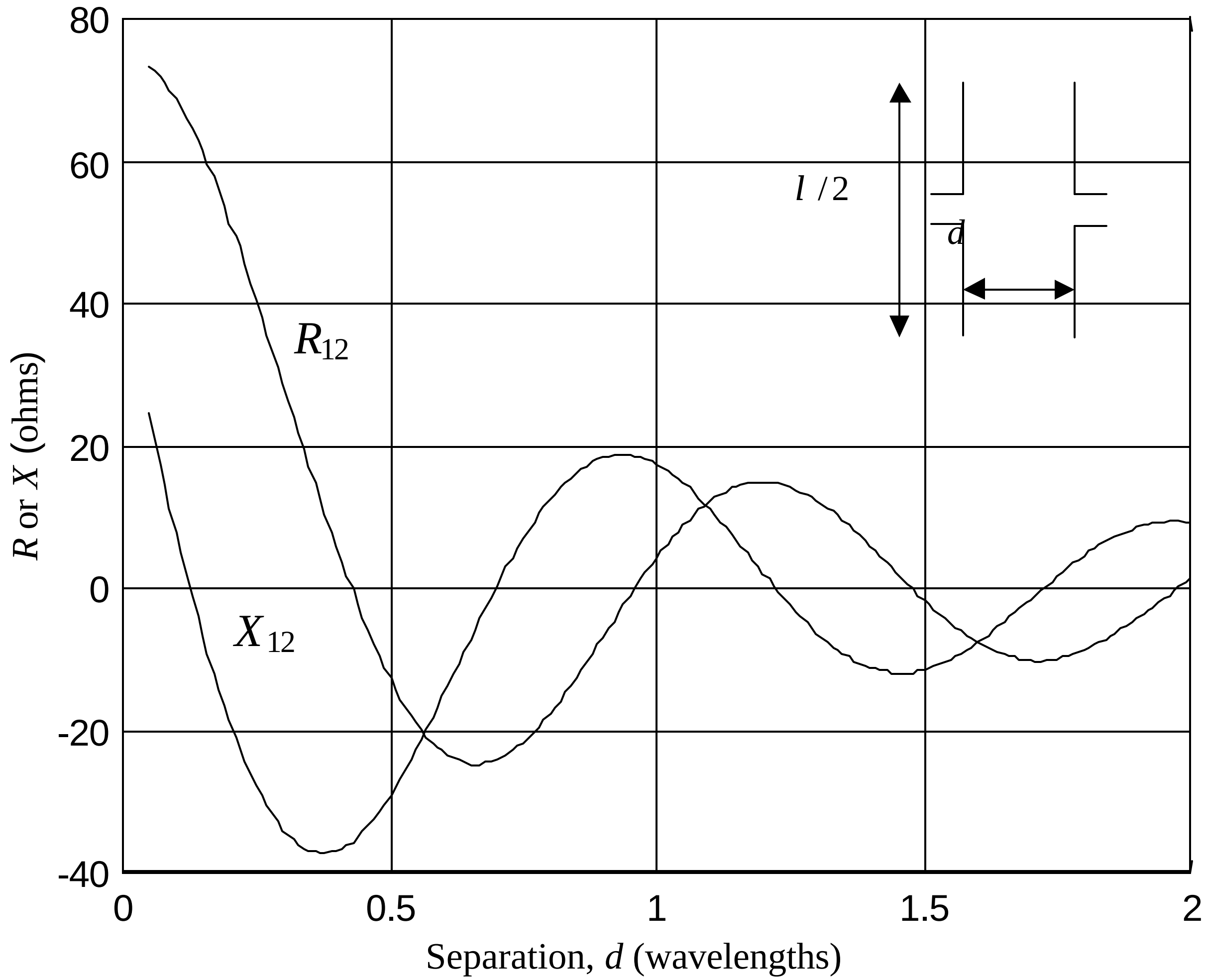
$$Z_{mn} = \left.\frac{V_m}{I_n}\right|_{I_m=0} = $$ mutual impedance at port m due to a current in port n, with port m open ciruited

By reciprocity this is the same as $Z_{mn} = Z_{nm} = \left.\frac{V_n}{I_m}\right|_{I_n=0}$. Say that we have two dipoles, one distant (n) and one near (m). To determine Z_{mn} a voltage can be applied to the distant dipole and the open-circuited current computed on the near dipole using the method of moments. The ratio of the distant dipole's voltage to the current induced on the near one gives the mutual impedance between the two dipoles.

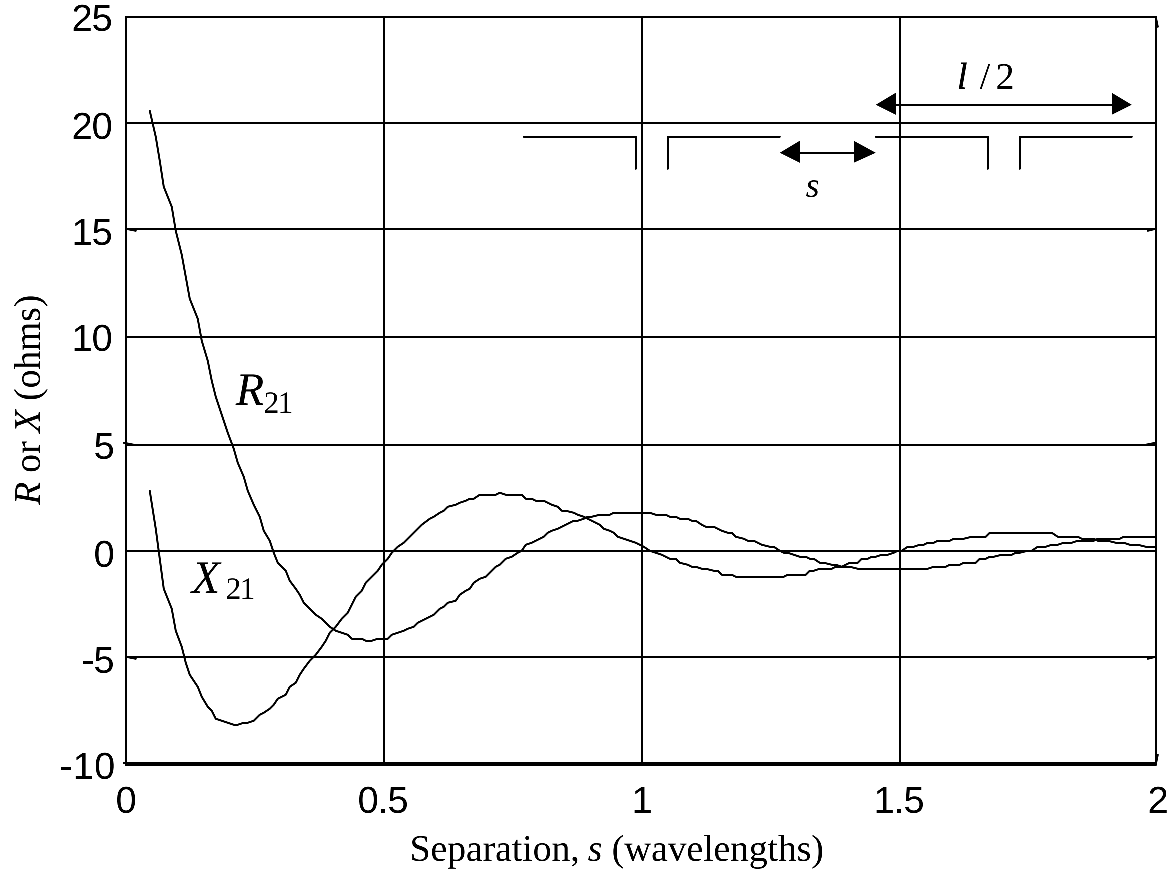
Plots of mutual impedance are shown on the following pages:

1. high mutual impedance implies strong coupling between the antennas
2. mutual impedance decreases with increasing separation between antennas
3. mutual impedance for wire antennas placed end to end is not as strong as when they are placed parallel

Mutual Impedance of Parallel Dipoles



Mutual Impedance of Colinear Dipoles



Mutual-Impedance Example

Example: Assume that a half wave dipole has been tuned so that it is resonant ($X_a = 0$). We found that a resonant half wave dipole fed by a 50 ohm transmission line has a VSWR of 1.46 (a reflection coefficient of 0.1870). If a second half wave dipole is placed parallel to the first one and 0.65 wavelength away, what is the input impedance of the first dipole?

From the plots, the mutual impedance between two dipoles spaced 0.65 wavelength is

$$Z_{21} = R_{21} + jX_{21} = -24.98 - j7.7\Omega$$

Noting that $Z_{21} = Z_{12}$ the total input impedance is

$$\begin{aligned} Z_{\text{in}} &= Z_{11} + Z_{21} \\ &= 73 + (-24.98 - j7.7) \\ &= 48.02 - j7.7 \ \Omega \end{aligned}$$

The reflection coefficient is

$$\Gamma = \frac{Z_{\text{in}} - Z_o}{Z_{\text{in}} + Z_o} = \frac{(48.02 - j7.7) - 50}{(48.02 - j7.7) + 50} = -0.0139 - j0.0797 = 0.081 e^{-j99.93^\circ}$$

which corresponds to a VSWR of 1.176. In this case the presence of the second dipole has improved the match at the input terminals of the first antenna.

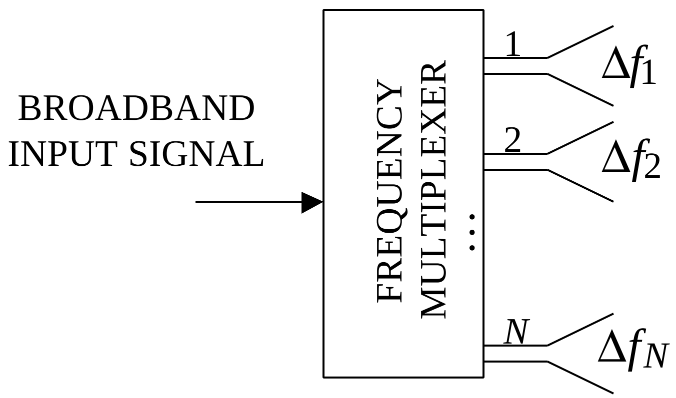
Broadband Antennas (1)

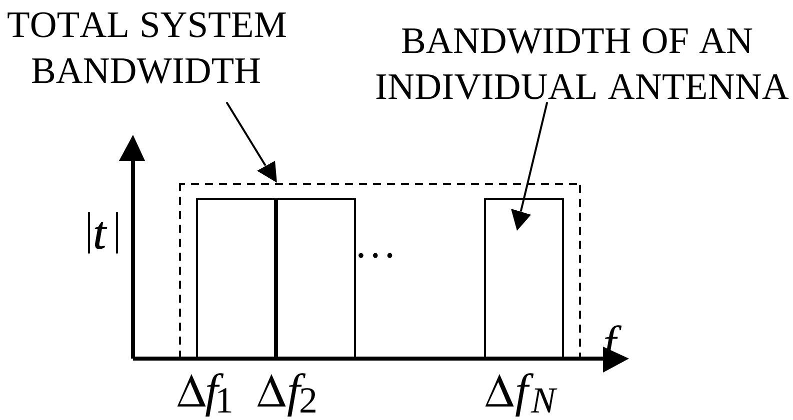
The required frequency bandwidth increases with the rate of information transfer (i.e., high data rates require wide frequency bandwidths). Designing an efficient wideband antenna is difficult. The most efficient antennas are designed to operate at a resonant frequency, which is inherently narrow band.

Two approaches to operating over wide frequency bands:

1. Split the entire band into sub-bands and use a separate resonant antenna in each band

Advantage: The individual antennas are easy to design (potentially inexpensive)
Disadvantage: Many antennas are required (may take a lot of space, weight, etc.)





Broadband Antennas (2)

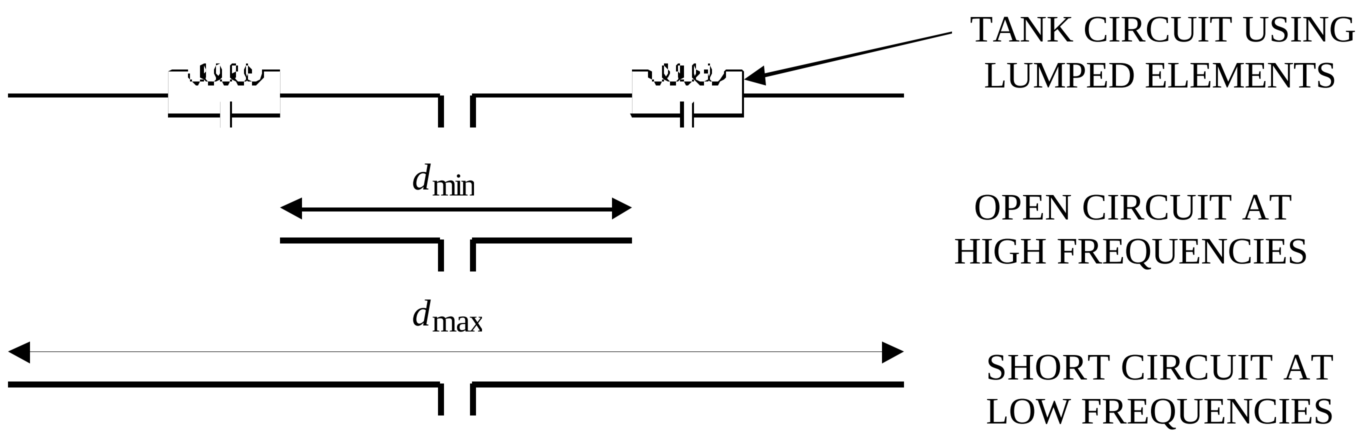
2. Use a single antenna that operates over the entire frequency band

Advantage: Less aperture area required by a single antenna
Disadvantage: A wideband antenna is more difficult to design than a narrowband antenna

Broadbanding of antennas can be accomplished by:

1. interlacing narrowband elements having non-overlapping sub-bands (stepped band approach)

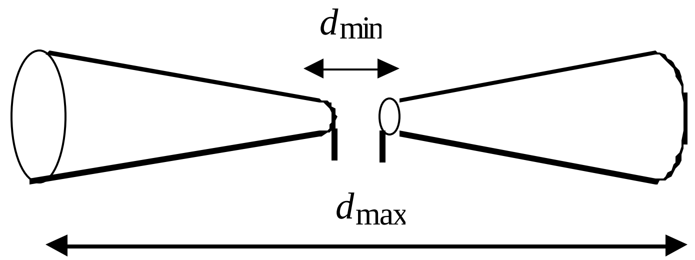
Example: multi-feed point dipole



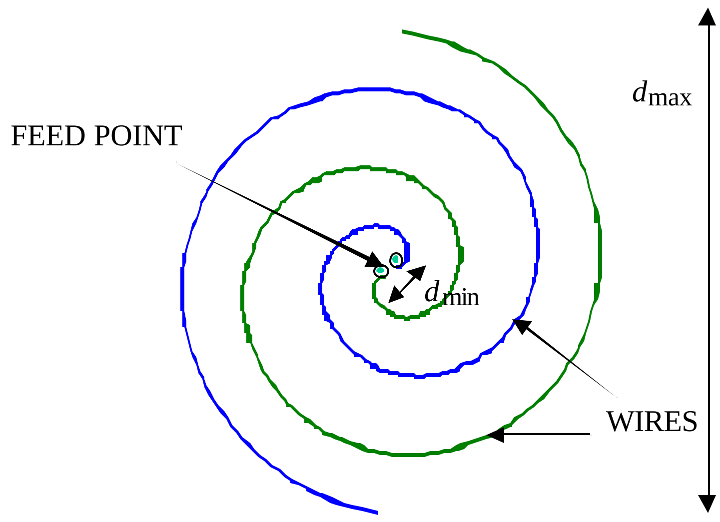
Broadband Antennas (3)

2. design elements that have smooth geometrical transitions and avoid abrupt discontinuities

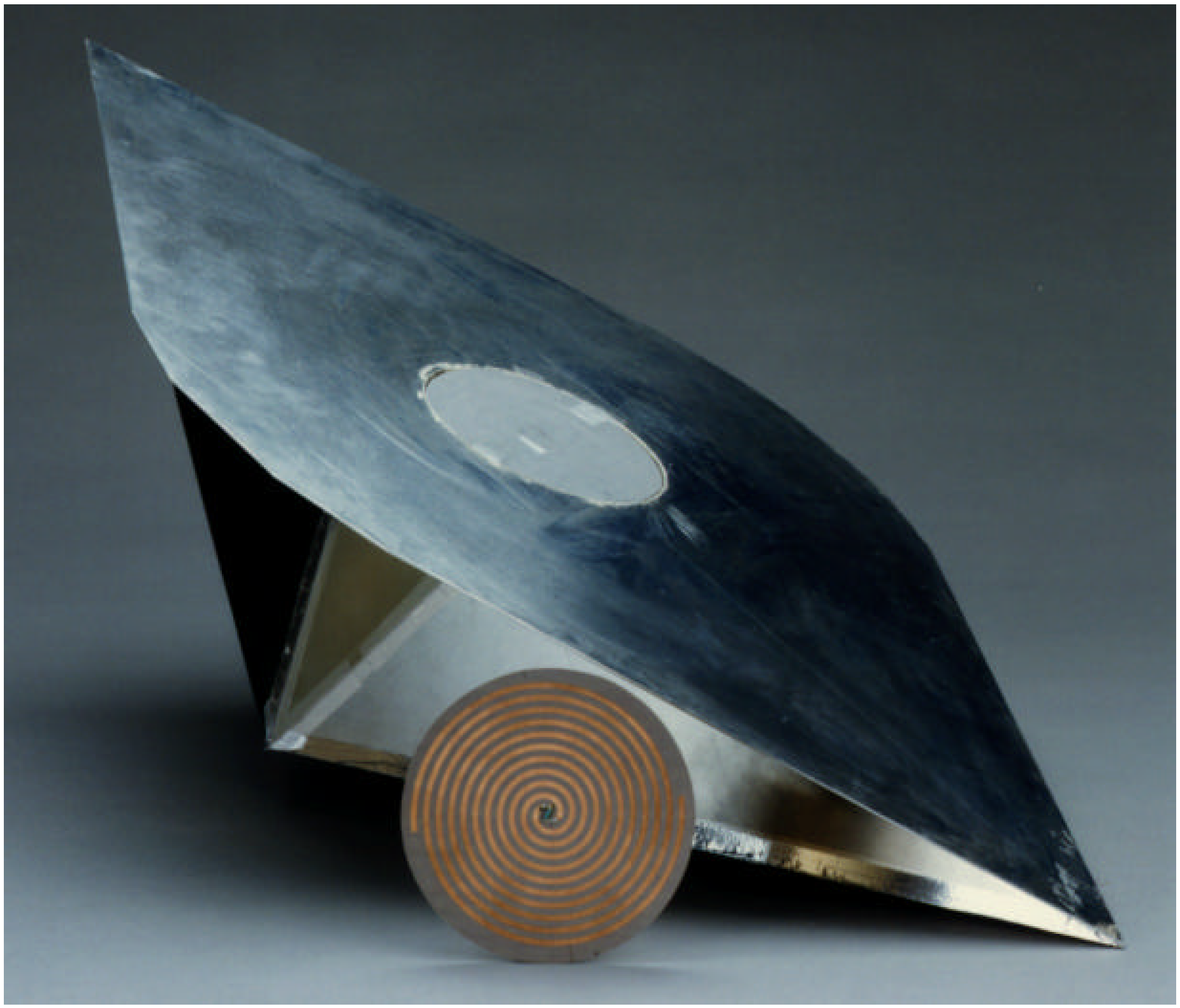
Example: biconical antenna



spiral antenna

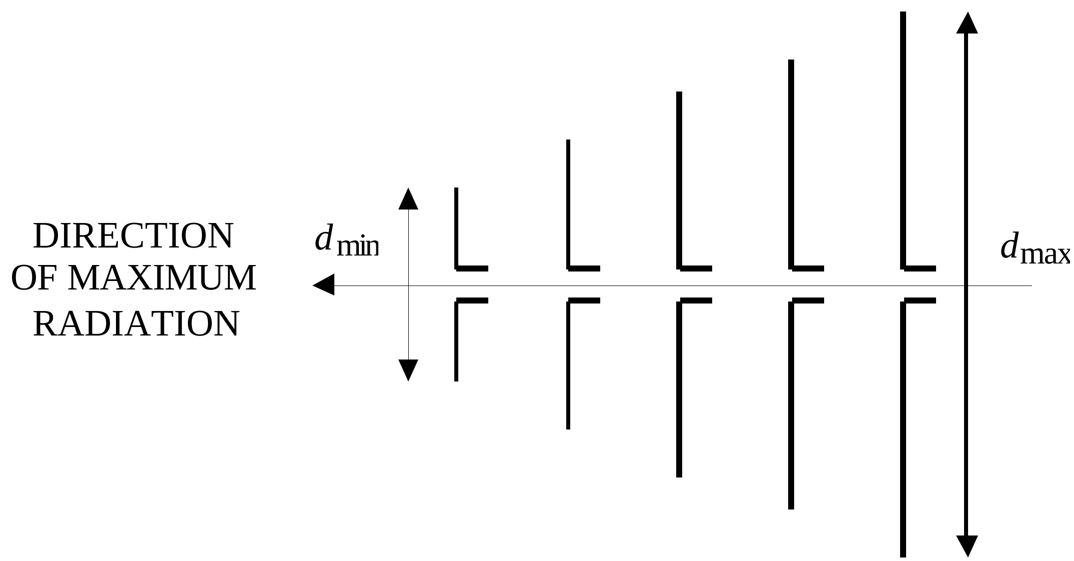


Circular Spiral in Low Observable Fixture



Broadband Antennas (4)

Another example of a broadband antenna is the log-periodic array. It can be classified as a single element with a gradual geometric transitions or as discrete elements that are resonant in sub-bands. All of the elements of the log periodic antenna are fed.

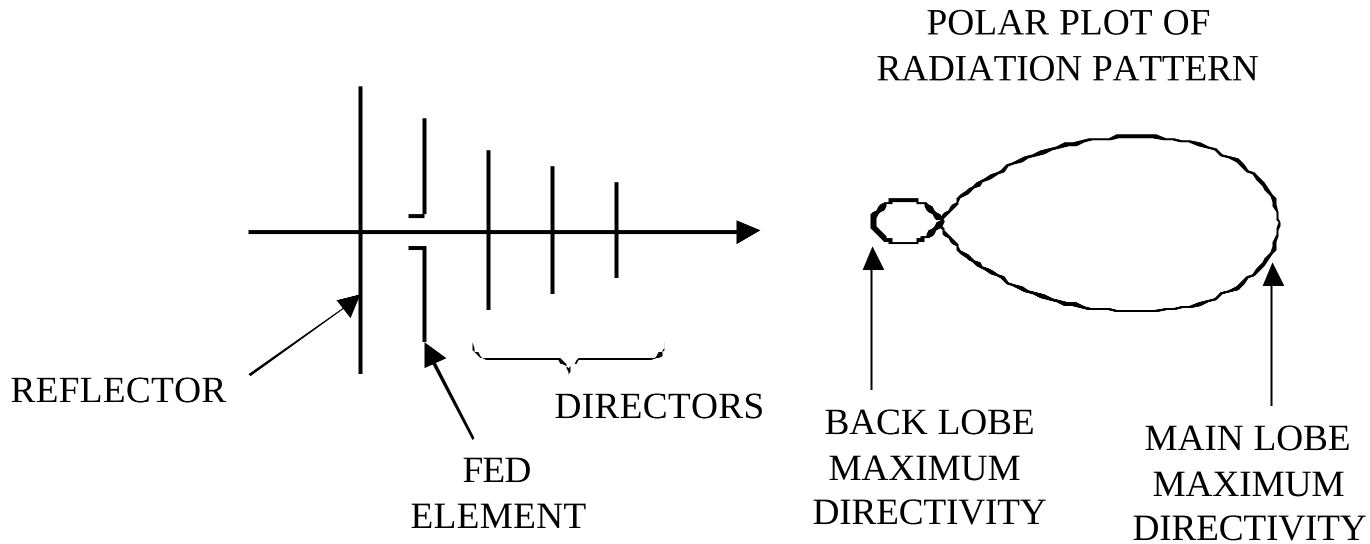


The range of frequencies over which ana antenna operates is determined approximately by the maximum and minimum antenna dimensions

$$d_{\min} \approx \frac{l_H}{2} \text{ and } d_{\max} \approx \frac{l_L}{2}$$

Yagi-Uda Antenna

A Yagi-Uda (or simply Yagi) is used at high frequencies (HF) to obtain a directional azimuth pattern. They are frequently employed as TV/FM antennas. A Yagi consists of a fed element and at least two parasitic (non-excited) elements. The shorter elements in the front are directors. The longer element in the back is a reflector. The conventional design has only one reflector, but may have up to 10 directors.

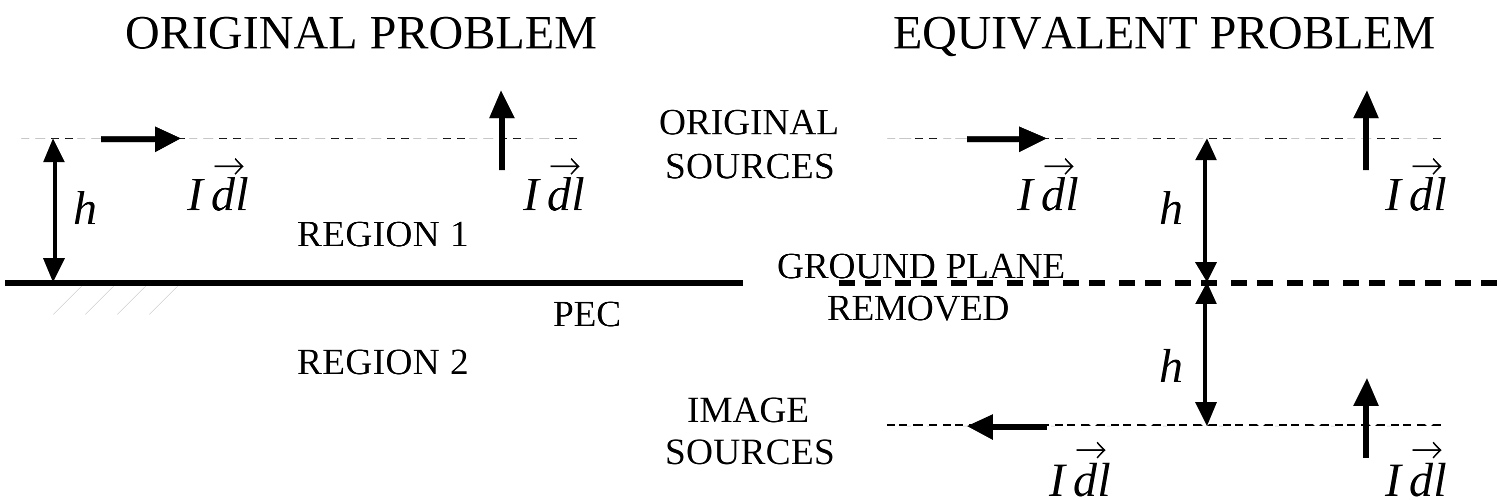


The front-to-back ratio is the ratio of the maximum directivity in the forward direction to that in the back direction: $D_{\max} / D_{\text{back}}$

Ground Planes and Images (1)

In some cases the method of images allows construction of an equivalent problem that is easier to solve than the original problem.

When a source is located over a PEC ground plane, the ground plane can be removed and the effects of the ground plane on the fields outside of the medium accounted for by an image located below the surface.



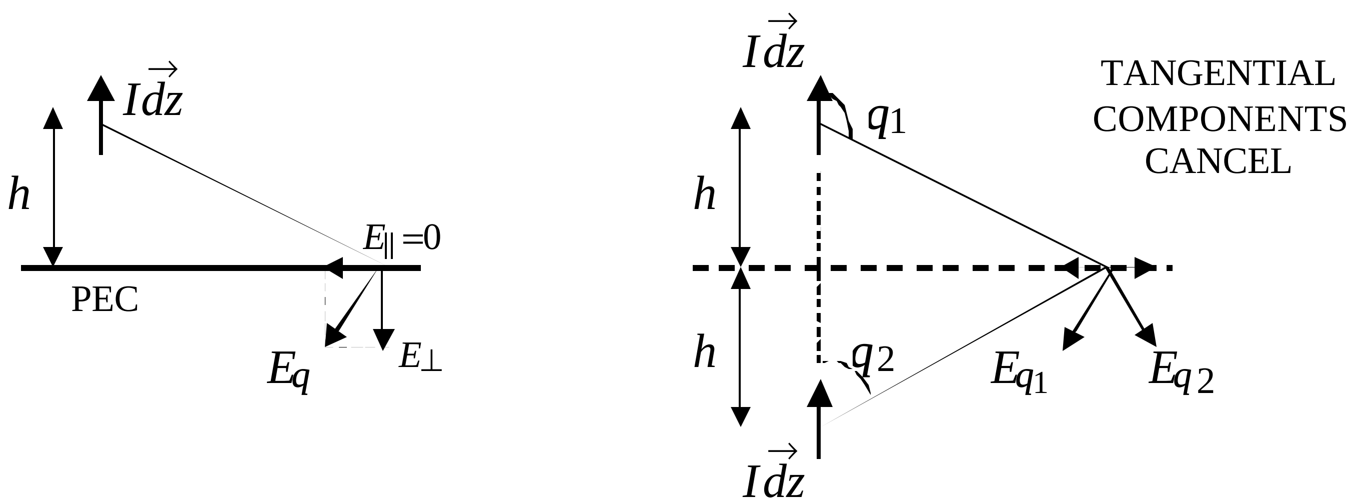
The equivalent problem holds only for computing the fields in region 1. It is exact for an infinite PEC ground plane, but is often used for finite, imperfectly conducting ground planes (such as the Earth's surface).

Ground Planes and Images (2)

The equivalent problem satisfies Maxwell's equations and the same boundary conditions as the original problem. The <u>uniqueness theorem</u> of electromagnetics assures us that the solution to the equivalent problem is the same as that for the original problem.

Boundary conditions at the surface of a PEC: the tangential component of the electric field is zero.

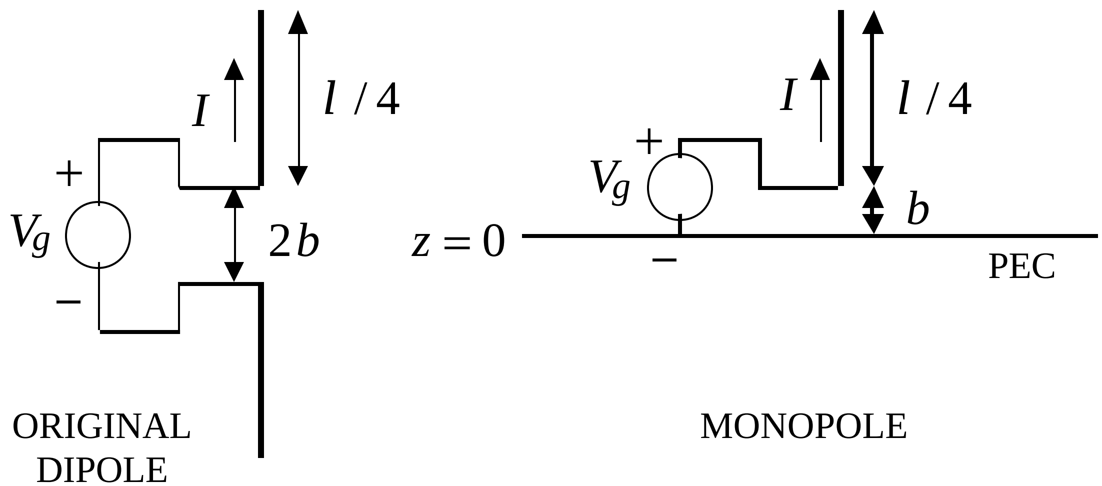
ORIGINAL PROBLEM EQUIVALENT PROBLEM



A similar result can be shown if the current element is oriented horizontal to the ground plane and the image is reversed from the source. (A reversal of the image current direction implies a negative sign in the image's field relative to the source field.)

Ground Planes and Images (3)

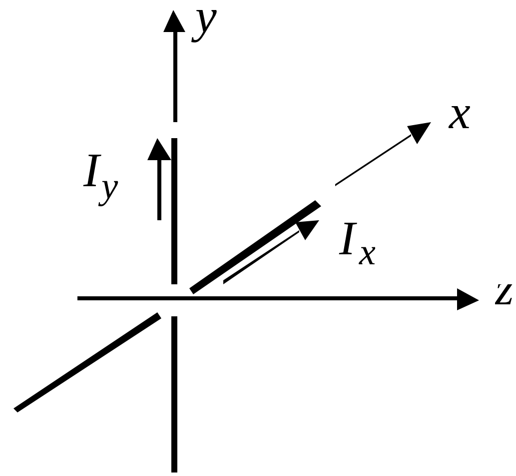
Half of a symmetric conducting structure can be removed if an infinite PEC is placed on the symmetry plane. This is the basis of a quarter-wave monopole antenna.



- The radiation pattern is the same for the monopole as it is for the half wave dipole above the plane $z = 0$
- The field in the monopole gap is twice the field in the gap of the dipole
- Since the voltage is the same but the gap is half of the dipole's gap

$$R_a\big|_{\text{monopole}} = \tfrac{1}{2} R_a\big|_{\text{dipole}} = \frac{73.12}{2} = 36.56 \text{ ohms}$$

Crossed Dipoles (1)



Crossed dipoles (also known as a turnstile) consists of two orthogonal dipoles excited 90 degrees out of phase.

$$I_x = I_o$$

$$I_y = I_o e^{jp/2} = jI_o$$

The radiation integral gives two terms

$$E_q = \frac{-jkh_o}{4pr} e^{-jkr}\left(\int_{-\ell/2}^{\ell/2} I_o \underbrace{\cos q \cos f}_{\hat{q}\bullet\hat{x}} e^{jx'k\sin q\cos f}\,dx' + j\int_{-\ell/2}^{\ell/2} I_o \underbrace{\cos q \sin f}_{\hat{q}\bullet\hat{y}} e^{jy'k\sin q\sin f}\,dy' \right)$$

If $k\ell << 1$ then $\int_{-\ell/2}^{\ell/2} e^{jx'k\sin q\cos f}\,dx' \approx \ell$ and similarly for the y integral. Therefore,

$$E_q = \underbrace{\frac{-jkhI_o\ell}{4p}}_{\equiv E_o} \frac{e^{-jkr}}{r}\cos q(\cos f + j\sin f) = E_o \frac{e^{-jkr}}{r}\cos q(\cos f + j\sin f)$$

Crossed Dipoles (2)

A similar result is obtained for E_f

$$E_f = \underbrace{\frac{jkhI_o \ell}{4p}}_{\equiv -E_o} \frac{e^{-jkr}}{r} (\sin f - j\cos f) = -E_o \frac{e^{-jkr}}{r} (\sin f - j \cos f)$$

Consider the components of the wave propagating toward an observer on the z axis $q = f = 0 : E_q = E_o, E_f = jE_o$, or

$$\vec{E} = E_o \frac{e^{-jkr}}{r} (\hat{x} + j\hat{y})$$

which is a circularly polarized wave. If the observer is not on the z axis, the projected lengths of the two dipoles are not equal, and therefore the wave is <u>elliptically polarized</u>. The <u>axial ratio</u> (AR) is a measure of the wave's ellipticity at the specified q, f:

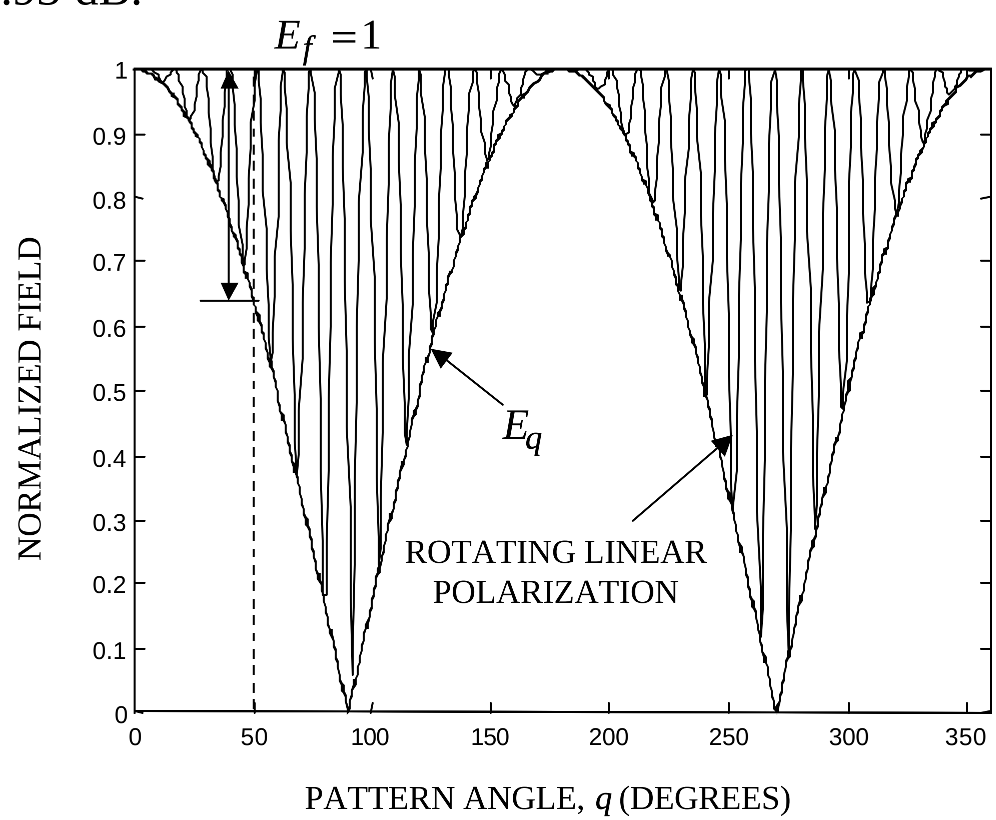
$$\mathrm{AR} = \frac{|E_{\mathrm{max}}|}{|E_{\mathrm{min}}|}, \quad 1 \le \mathrm{AR} \le \infty$$

For the crossed dipoles

$$\mathrm{AR} = \frac{|E_f|}{|E_q|} = \frac{1}{\sqrt{\cos^2 q (\cos^2 f + \sin^2 f)}} = \frac{1}{|\cos q|}$$

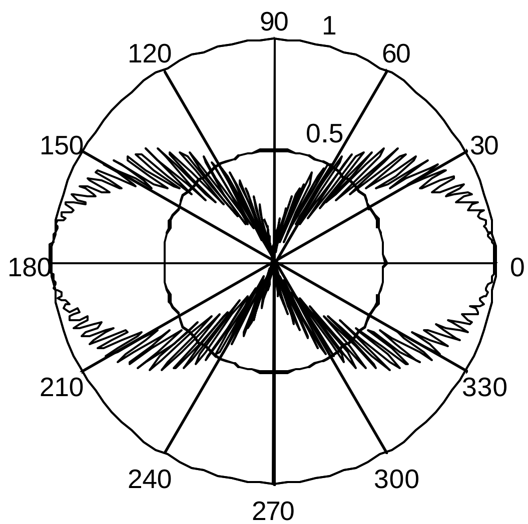
Crossed Dipoles (3)

The rotating linear pattern is shown. A linear receive antenna rotates like a propeller blade as it measures the far field at range r. The envelope of the oscillations at any particular angle gives the axial ratio at that angle. For example, at 50 degrees the AR is about $1/0.64 = 1.56 = 1.93$ dB.

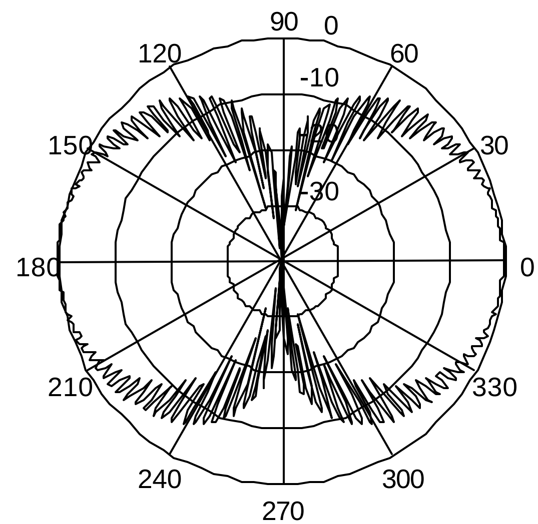


Crossed Dipoles (4)

Examples of rotating linear patterns on crossed dipoles that are not equal in length



VOLTAGE PLOT

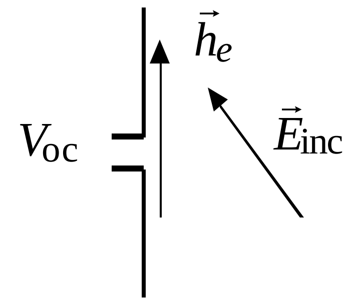


DECIBEL PLOT

Polarization Loss (1)

For linear antennas an effective height ($\vec{h}_e$) can be defined

$$V_{oc} = \vec{E}_{inc} \bullet \vec{h}_e$$



The open circuit voltage is a maximum when the antenna is aligned with the incident electric field vector. The effective height of an arbitrary antenna can be determined by casting its far field in the following form of three factors

$$\vec{E}(r,\theta,\phi) = [E_o]\left[\frac{e^{-jkr}}{r}\right]\left[\vec{h}_e(\theta,\phi)\right]$$

The effective height accounts for the incident electric field projected onto the antenna element. The polarization loss factor (PLF) between the antenna and incident field is

$$\text{PLF}, p = \frac{\left|\vec{E}_{inc} \bullet \vec{h}_e\right|^2}{\left|\vec{E}_{inc}\right|^2 \left|\vec{h}_e\right|^2}$$

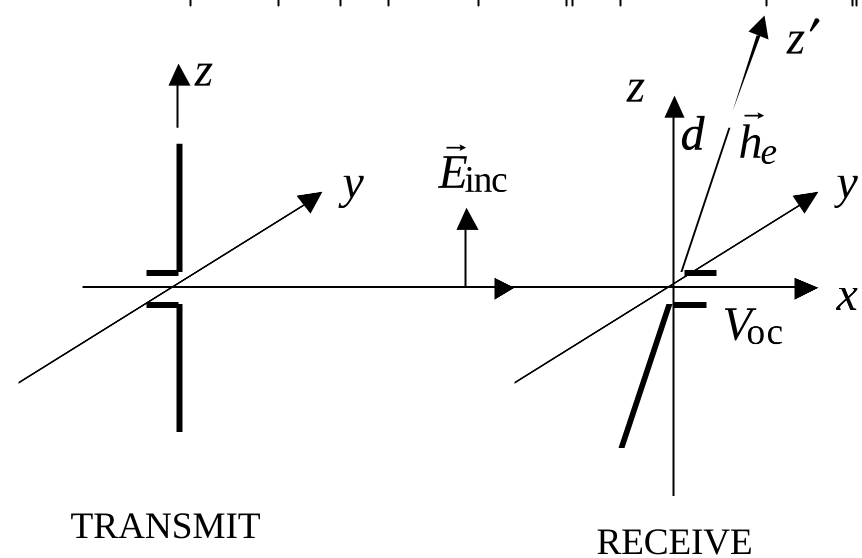
Polarization Loss (2)

Example: The Hertzian dipole's far field is

$$\vec{E}(r,q,f)=\left[\frac{jhkI_o}{4p}\right]\left[\frac{e^{-jkr}}{r}\right]\underbrace{\left[\ell\sin q\,\hat{q}\right]}_{\vec{h}_e(q,f)}$$

If we have a second dipole that is rotated by an angle d in a plane parallel to the plane containing the first dipole, we can calculate the PLF as follows. First,

$$V_{\text{oc}}=\vec{E}_{\text{inc}}\bullet\vec{h}_e=\left|\vec{E}_{\text{inc}}\right|\hat{z}\bullet\left|\vec{h}_e\right|\hat{z}'=\left|\vec{E}_{\text{inc}}\right|\left|\vec{h}_e\right|\hat{z}\bullet\hat{z}'=\left|\vec{E}_{\text{inc}}\right|\left|\vec{h}_e\right|\cos d$$



Polarization Loss (3)

The PLF is

$$p = \frac{\left|\vec{E}_{\text{inc}} \bullet \vec{h}_e\right|^2}{\left|\vec{E}_{\text{inc}}\right|^2 \left|\vec{h}_e\right|^2} = \frac{\left|\vec{E}_{\text{inc}}\right|^2 \left|\vec{h}_e\right|^2 \cos^2 d}{\left|\vec{E}_{\text{inc}}\right|^2 \left|\vec{h}_e\right|^2} = \cos^2 d$$

When the dipoles are parallel, p=1, and there is no loss due to polarization mismatch. However, when the dipoles are at right angles, p=0 and there is a complete loss of signal.

A more general case occurs when the incident field has both q and f components
$\vec{E}_{\text{inc}} = E_{i_q}\hat{q} + E_{i_f}\hat{f}$

$$p = \frac{\left|\left(E_{i_q}\hat{q} + E_{i_f}\hat{f}\right) \bullet \vec{h}_e\right|^2}{\left|E_{i_q}\hat{q} + E_{i_f}\hat{f}\right|^2 \left|\vec{h}_e\right|^2}$$

Example: The effective height of a RHCP antenna which radiates in the +z direction is given by the vector $\vec{h}_e = h_o\left(\hat{q} - j\hat{f}\right)$. A LHCP field is incident on this antenna (i.e., the incident wave propagates in the –z direction):

$$\vec{E}_{\text{inc}} = E_o\left(\hat{q} - j\hat{f}\right)e^{jkz}$$

Polarization Loss (4)

The PLF is

$$p = \frac{\left| E_o h_o \left(\hat{q} - j\hat{f}\right) \bullet \left(\hat{q} - j\hat{f}\right) e^{jkz} \right|^2}{\left|\sqrt{2} E_o\right|^2 \left|\sqrt{2} h_o\right|^2} = 0$$

If a RHCP wave is incident on the same antenna, again propagating along the *z* axis in the negative direction, $\vec{E}_{\text{inc}} = E_o \left(\hat{q} + j\hat{f}\right) e^{jkz}$. Now the PLF is

$$p = \frac{\left| E_o h_o \left(\hat{q} - j\hat{f}\right) \bullet \left(\hat{q} + j\hat{f}\right) e^{jkz} \right|^2}{\left|\sqrt{2} E_o\right|^2 \left|\sqrt{2} h_o\right|^2} = 1$$

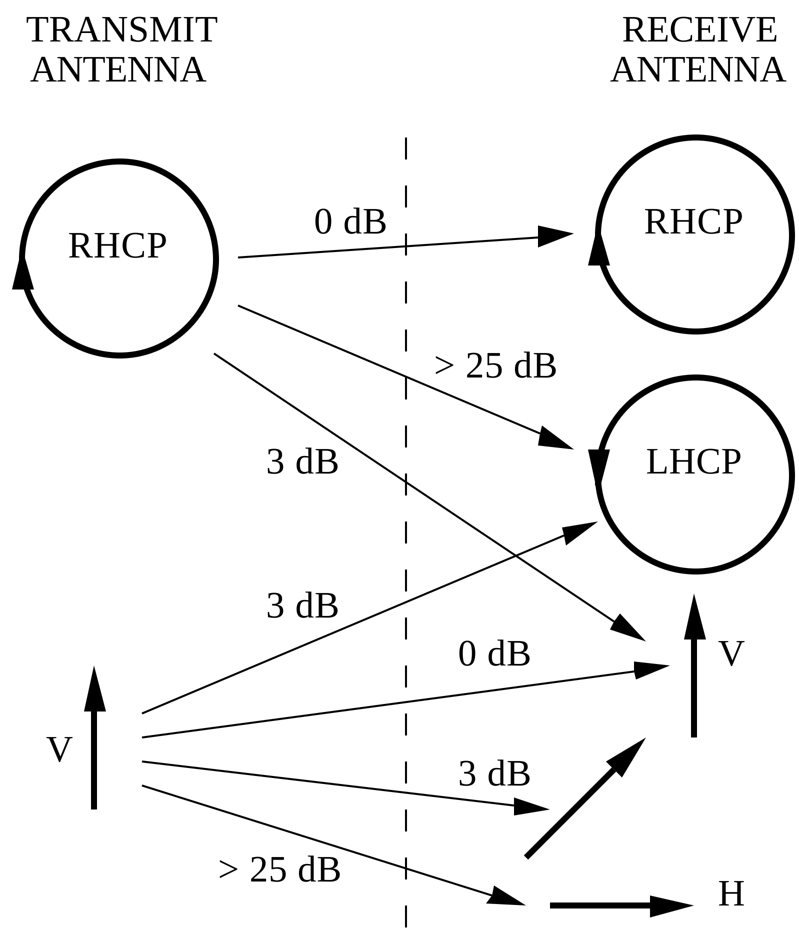
Finally, if a linearly polarized plane wave is incident on the antenna, $\vec{E}_{\text{inc}} = \hat{q} E_o e^{jkz}$

$$p = \frac{\left| E_o h_o \left(\hat{q} - j\hat{f}\right) \bullet \hat{q} e^{jkz} \right|^2}{\left| E_o\right|^2 \left|\sqrt{2} h_o\right|^2} = 1/2$$

If a linearly polarized antenna is used to receive a circularly polarized wave (or the reverse situation), there is a 3 dB loss in signal.

Antenna Polarization Loss

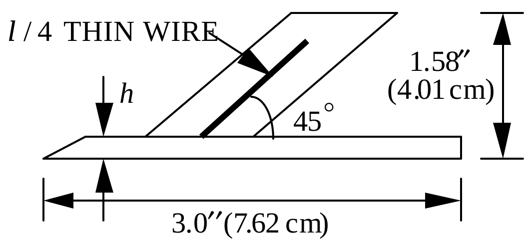
Summary of polarization losses for polarization mismatched antennas



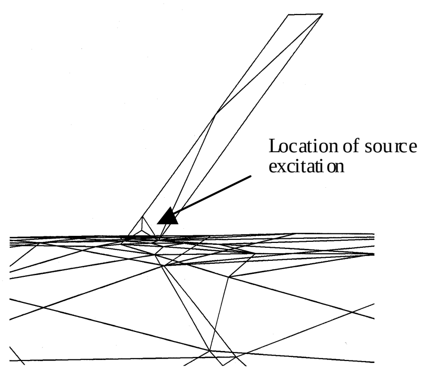
Aircraft Blade Antenna

Blade antennas are used for telemetry and communications. They have nearly hemispherical coverage, allowing the aircraft to maneuver without a complete loss of signal. The resulting polarization is often called slant because it contains both horizontal and vertical components.

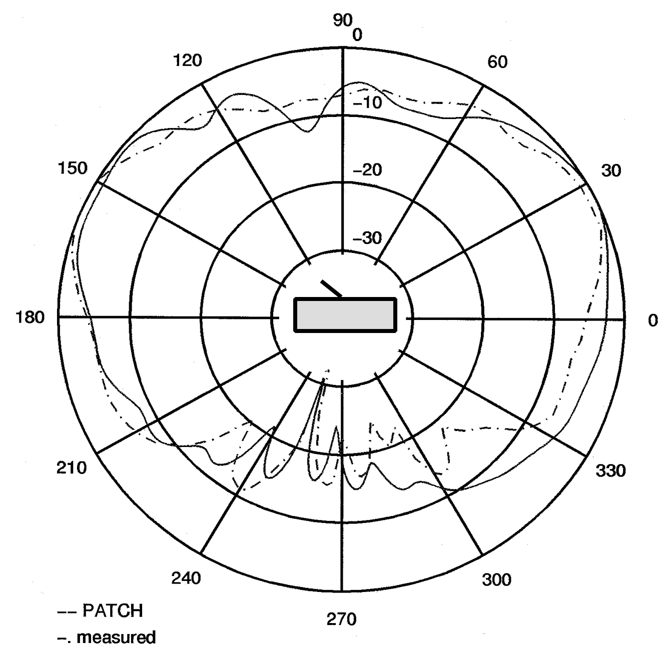
Blade antenna



Method of moments model:



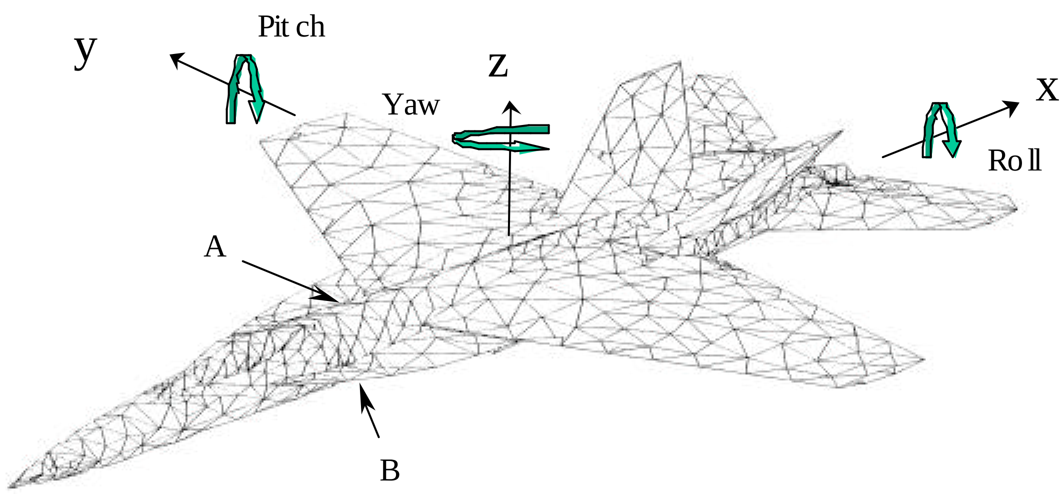
Comparison of measured and calculated patterns for a blade installed on a cylinder



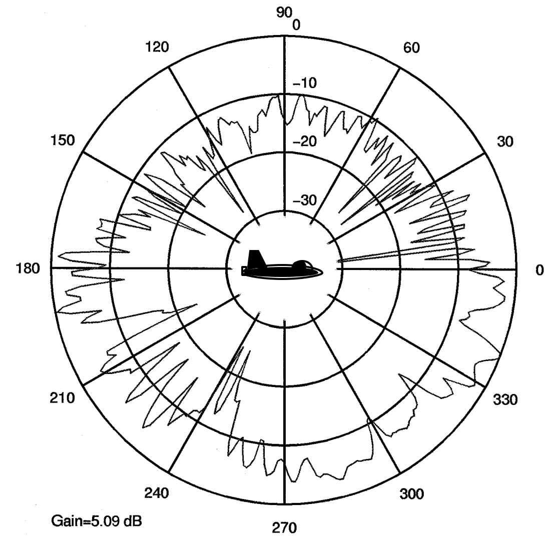
Blade Antennas Installed on an Aircraft

The top antenna (A) provides coverage in the upper hemisphere, while the bottom antenna (B) covers the lower hemisphere. The two antennas can be duplexed (switched) or their signals combined using a coupler.

Method of moments patch model:



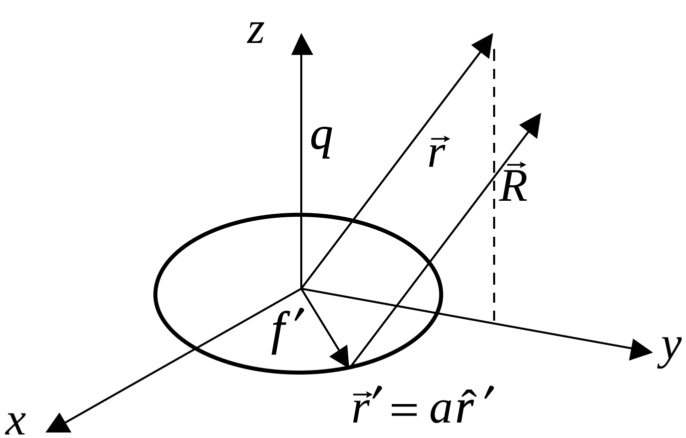
Elevation pattern: (signals combined with 3 dB coupler)



Small Loop Antenna (1)

By symmetry, we expect that the field of a small wire loop located in the $z = 0$ plane will depend only on the angle off of the wire axis, q . Because of the azimuthal symmetry cylindrical coordinates are required to solve this problem. If the wire is very thin (a filament) and has a constant current $I_o\hat{f}'$ flowing in it, the radiation integral is

$$\vec{E}(r,q,f) \approx \frac{-jkh}{4pr} e^{-jkr} \int_0^{2p} I_o\, e^{jk(\vec{r}'\bullet\hat{r})} \underbrace{\hat{f}' a\, df'}_{d\vec{l}'}$$



Using the transformation tables

$$\hat{r} = \hat{x}\sin q\,\cos f + \hat{y}\sin q\,\sin f + \hat{z}\cos q$$

$$\vec{r}' = a\hat{r}' = a\left(\hat{x}\cos f' + \hat{y}\sin f'\right)$$

$$\hat{r}\bullet\vec{r}' = a\sin q\left(\cos f\,\cos f' + \sin f\,\sin f'\right)$$

We also need $\hat{f}'$ in terms of the cartesian unit vectors ($\hat{f}'$ is <u>not</u> a constant that can be moved outside of the integral)

$$\hat{f}' = -\hat{x}\sin f' + \hat{y}\cos f'$$

Small Loop Antenna (2)

The radiation integral becomes

$$\vec{E}(r,q,f) \approx \frac{-jkhI_o a}{4pr} e^{-jkr} \int_0^{2p} \left(-\hat{x}\sin f' + \hat{y}\cos f'\right) e^{jka\sin q\,(\cos f\,\cos f' + \sin f\,\sin f')} df'$$

For a small loop $ka << 1$ and the exponential can be represented by the first two terms of a Taylor's series to get $e^{jka\sin q\,(\cos f\,\sin f' + \sin f\,\cos f')} \approx 1 + jka\sin q\left(\cos f\,\cos f' + \sin f\,\sin f'\right)$
Inserting the approximation in the integral

$$\int_0^{2p} \left(-\hat{x}\sin f' + \hat{y}\cos f'\right)\left[1 + jka\sin q\,\left(\cos f\,\cos f' + \sin f\,\sin f'\right)\right] df'$$

Since $\int_0^{2p} \sin f'\,df' = \int_0^{2p} \cos f'\,df' = 0$ the 1 in the square brackets can be dropped. The remaining terms in the integrand involve the following factors:

$\sin f\,\sin^2 f'$

$\sin f'\cos f\,\cos f' \quad \rightarrow$ integrates to zero because $\sin f'\cos f'$ is an odd function of f'

$\cos f\,\cos^2 f'$

$\sin f'\sin f\,\cos f' \quad \rightarrow$ integrates to zero because $\sin f'\cos f'$ is an odd function of f'

Small Loop Antenna (3)

The only two terms that do not integrate to zero are of the form

$$\int_0^{2p} \sin^2 f' df' = \int_0^{2p} \cos^2 f' df' = p$$

Therefore,

$$\vec{E}(r,q,f) \approx \frac{-jkahI_o\left(jka\sin q\right)}{4pr} e^{-jkr}\left(-\hat{x}\sin f \int_0^{2p} \sin^2 f' df' + \hat{y}\cos f \int_0^{2p} \cos^2 f' df'\right)$$

$$= \frac{k^2hI_oa^2}{4r}\sin q \; e^{-jkr}\underbrace{\left(-\hat{x}\sin f + \hat{y}\cos f\right)}_{=\hat{f}} = \hat{f}\,\frac{k^2hI_oa^2}{4r}\sin q \; e^{-jkr}$$

The radiation pattern of the small loop is the same as that of a short dipole aligned with the loop axis. The radiated power is

$$P_{\text{rad}} = \frac{k^4 h I_o^2 p a^4}{12}$$

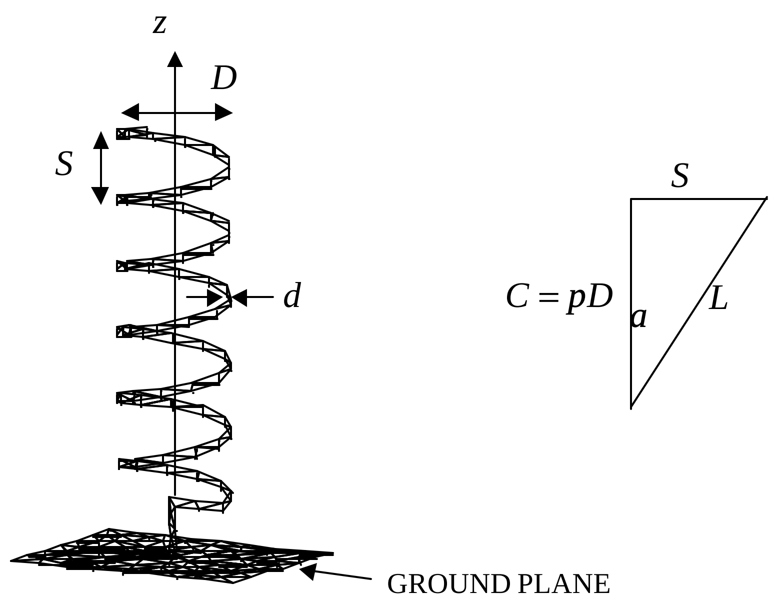
and the radiation resistance

$$R_a = \frac{2P_{\text{rad}}}{I_o^2} = \frac{k^4 h p a^4}{6} = \frac{(2p/l)^4 (120p) p a^4}{6} = 320p^4\left(\frac{a}{l}\right)^4$$

Helix Antenna (1)

A helix is described by the following parameters:

D = diameter
A = axial length
C = circumference (pD)
S = center-to-center spacing between turns
L = length of one turn
N = number of turns
$a = \tan^{-1}(S/pD) =$ pitch angle

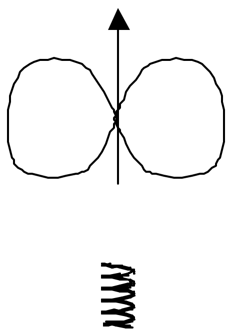


Conventional helices are constructed with air or low-dielectric cores. A helix is capable of operating in several different radiation modes and polarizations, depending on the combination of parameter values and the frequency.

Helix Antenna (2)

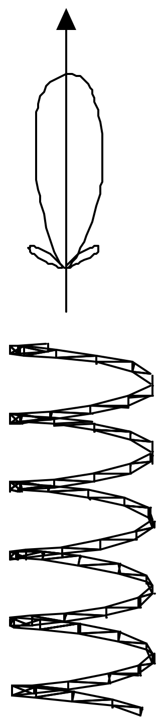
Radiation modes of a helix:

NORMAL MODE



SMALL (LOOKS LIKE A LOOP)

AXIAL MODE



LARGE

$C > l$

In both cases the radiation is circularly polarized:

Normal mode: $\text{AR} = \dfrac{2S\,l}{D^2 p^2}$

Axial mode: $\text{AR} = \dfrac{2n+1}{2n}$

In the axial mode, the beamwidth decreases with increasing helix length, *NS*

$$\text{BWFN} = \frac{115^\circ}{(C/l)\sqrt{NS/l}}$$

for $12^\circ < a < 13^\circ$.